Uczenie maszynowe w bioinformatyce

Wykład 10: sztuczne sieci neuronowe typu MLP

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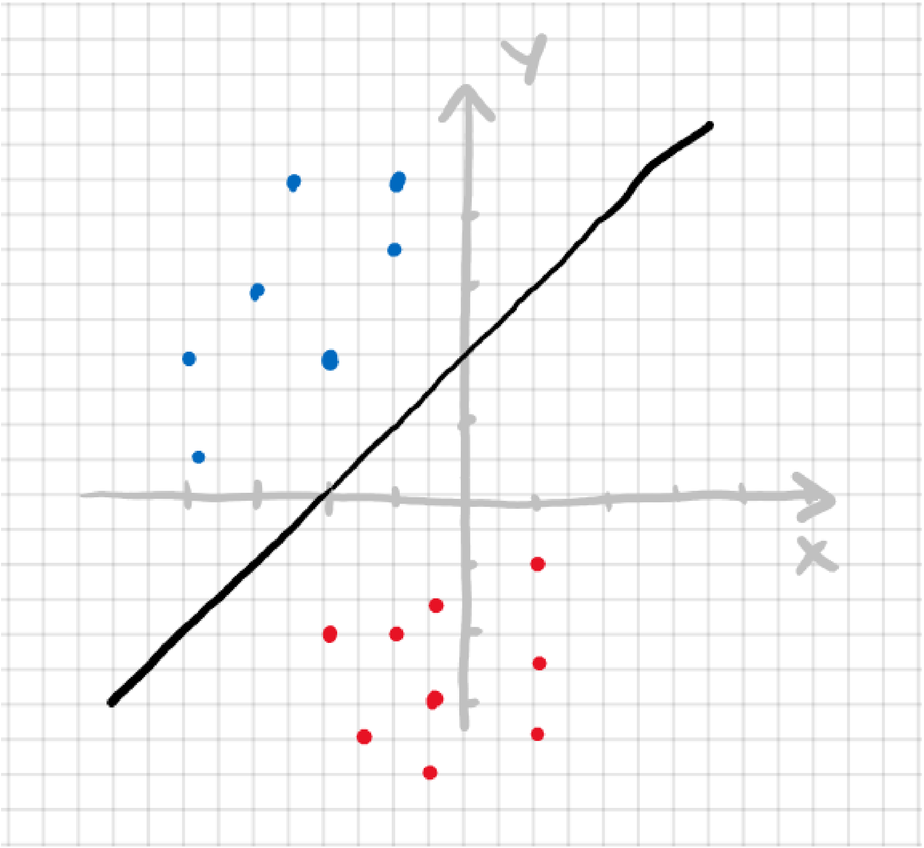
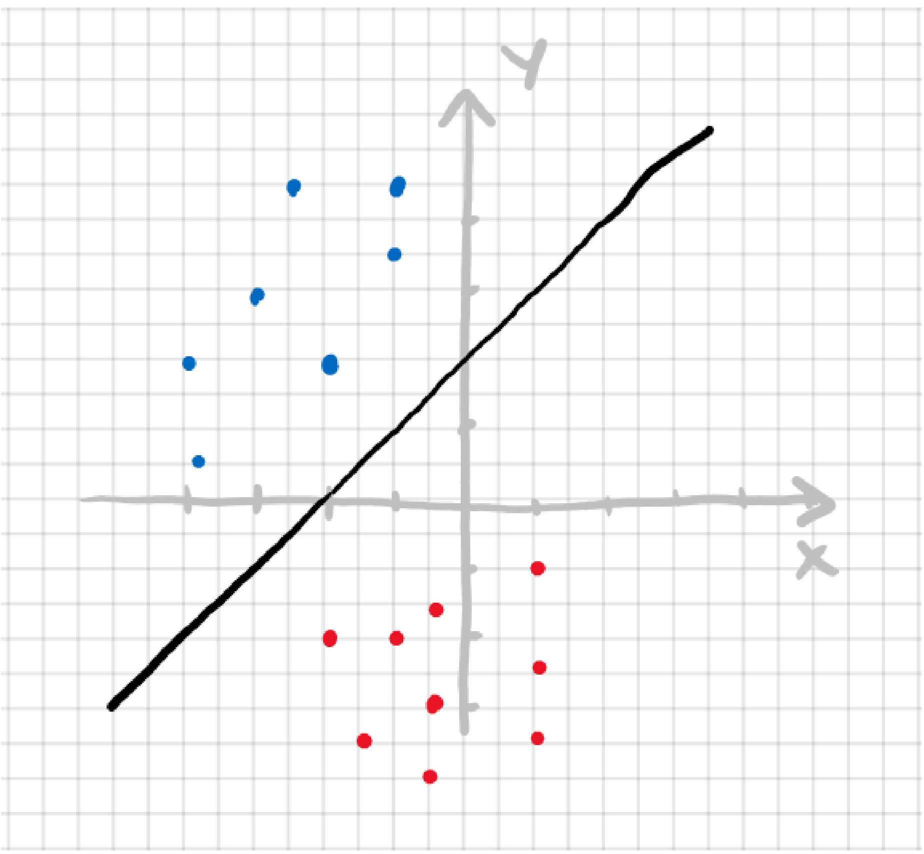
Klasyfikator liniowy

$y = ax + b$



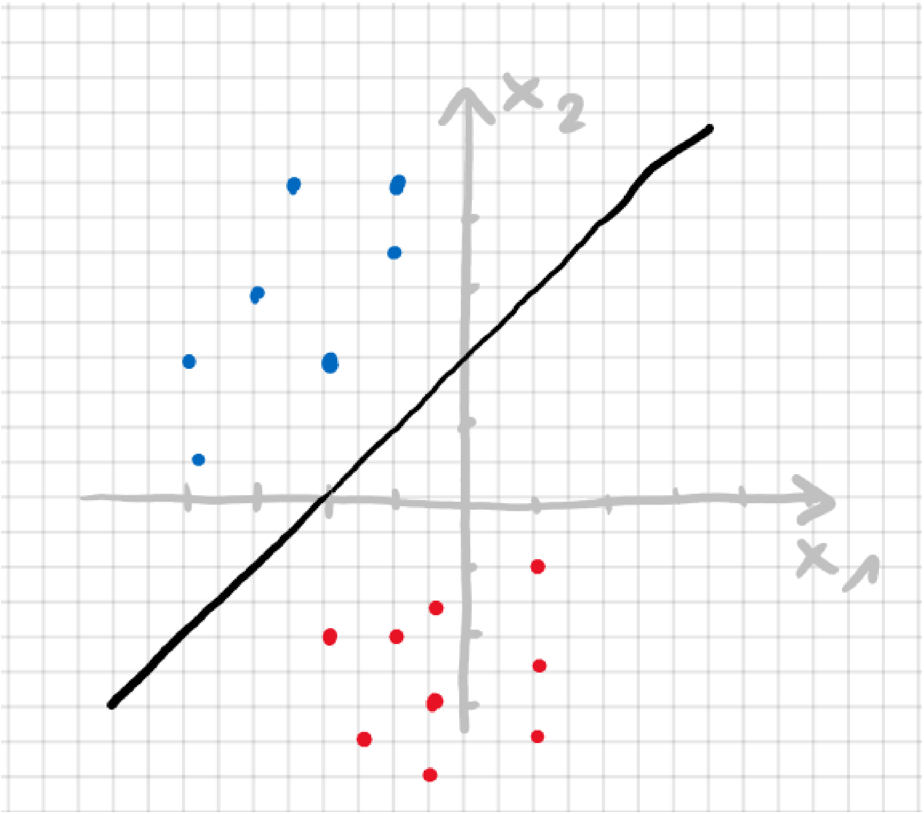
Klasyfikator liniowy

$$a x - y + b = 0$$



Klasyfikator liniowy

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$



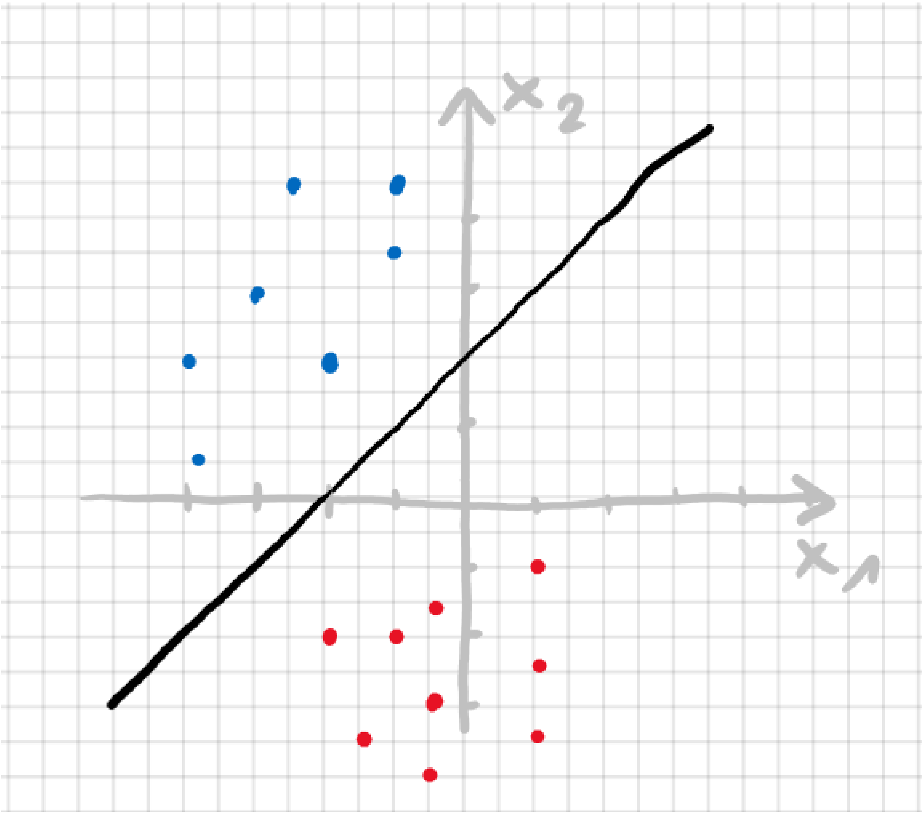
Klasyfikator liniowy

$$\underbrace{w_1 x_1 + w_2 x_2 + w_0 = 0}_{w^T \cdot x + w_0 = 0}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$w_0$$



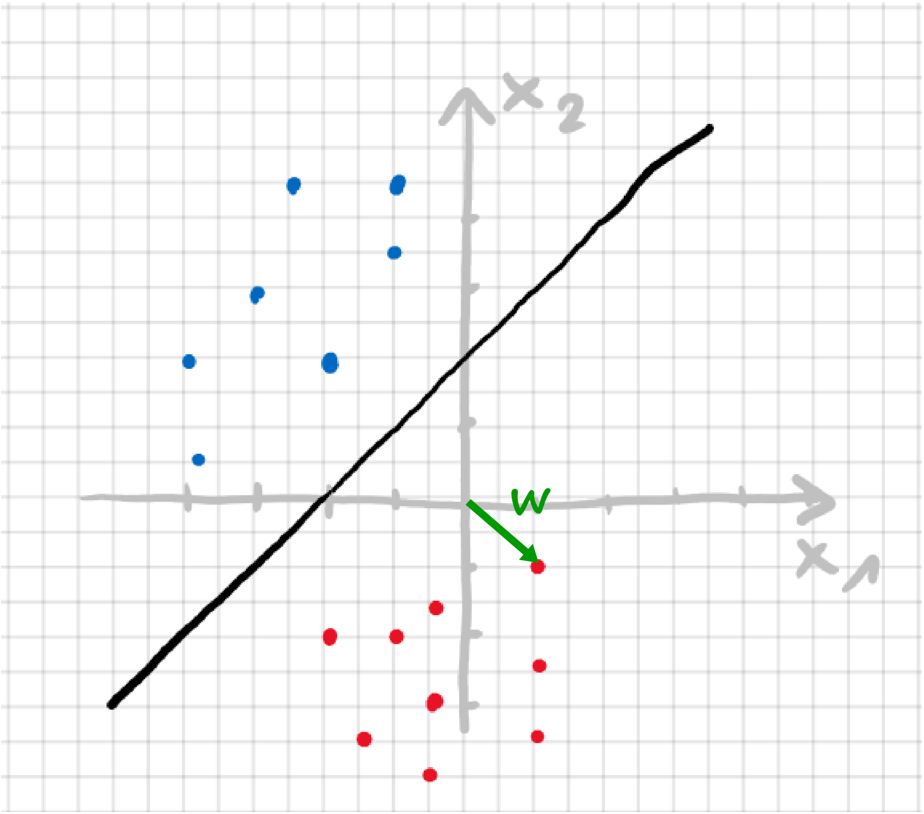
Klasyfikator liniowy

$$\underbrace{w_1 x_1 + w_2 x_2 + w_0 = \emptyset}_{w^T \cdot x + w_0 = \emptyset}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$w_0 = 2$$



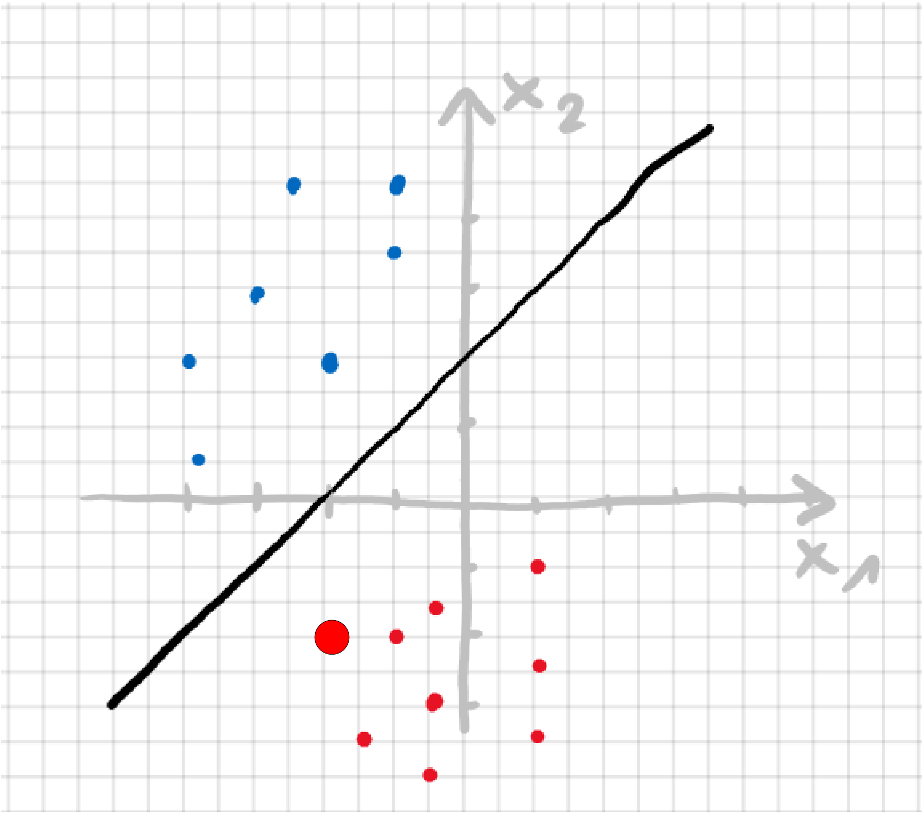
Klasyfikator liniowy

$$\underbrace{w_1 x_1 + w_2 x_2 + w_0 = 0}_{w^T \cdot x + w_0 = 0}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$w_0 = 2$$



$$\delta(x) = w^T \cdot x + w_0 = w_1 x_1 + w_2 x_2 + w_0 \Rightarrow \delta(x) = 1 \cdot (-2) + (-1) \cdot (-2) + 2 = 2$$

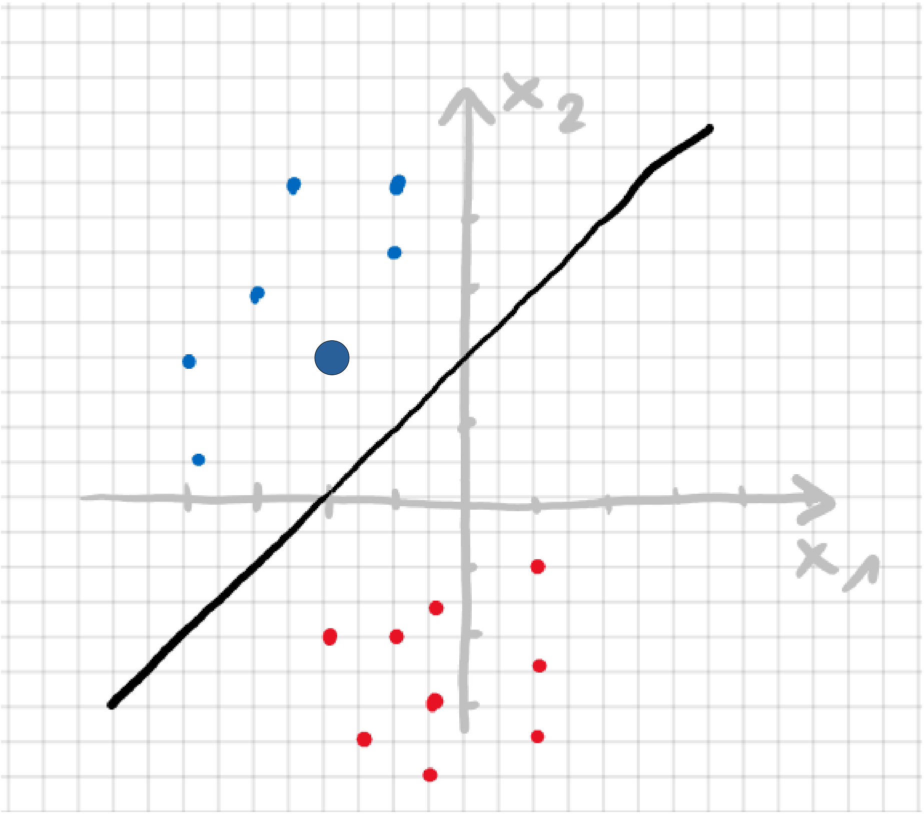
Klasyfikator liniowy

$$\underbrace{w_1 x_1 + w_2 x_2 + w_0 = 0}_{w^T \cdot x + w_0 = 0}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$w_0 = 2$$



$$\delta(x) = w^T \cdot x + w_0 = w_1 x_1 + w_2 x_2 + w_0 \Rightarrow \delta(x) = 1 \cdot (-2) + (-1) \cdot 2 + 2 = -2$$

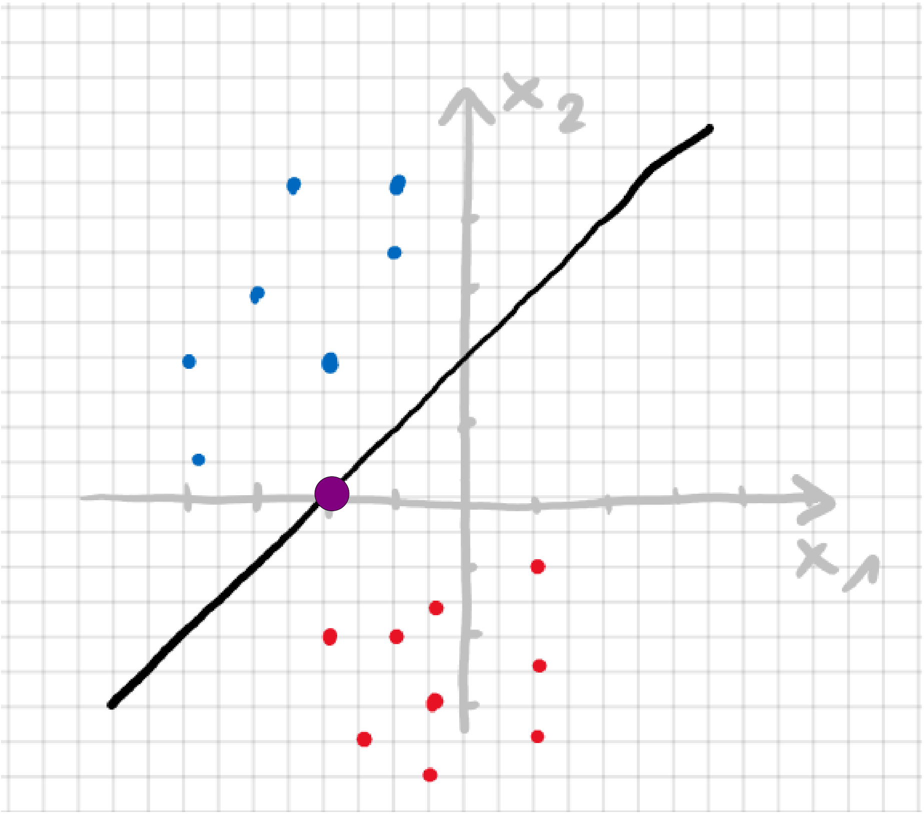
Klasyfikator liniowy

$$\underbrace{w_1 x_1 + w_2 x_2 + w_0 = 0}_{w^T \cdot x + w_0 = 0}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$w_0 = 2$$



$$\delta(x) = w^T \cdot x + w_0 = w_1 x_1 + w_2 x_2 + w_0 \Rightarrow \delta(x) = 1 \cdot (-2) + (-1) \cdot 0 + 2 = 0$$

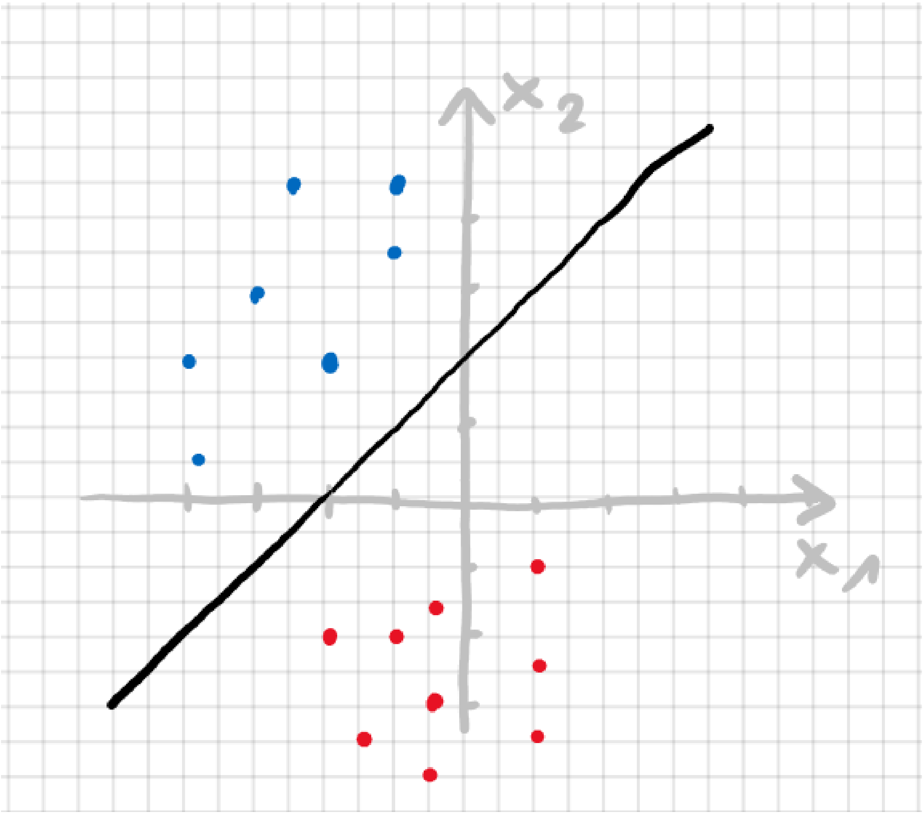
Klasyfikator liniowy

$$\underbrace{w_1 x_1 + w_2 x_2 + w_0 = \emptyset}_{w^T \cdot x + w_0 = \emptyset}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$w_0$$



Ogólna postać klasyfikatora liniowego

$$\delta(x) = w^T \cdot x + w_0$$

$$d(x) = \operatorname{sign}(\delta(x)) = \begin{cases} 1 & \Leftrightarrow \delta(x) > \emptyset \\ -1 & \Leftrightarrow \delta(x) < \emptyset \end{cases}$$

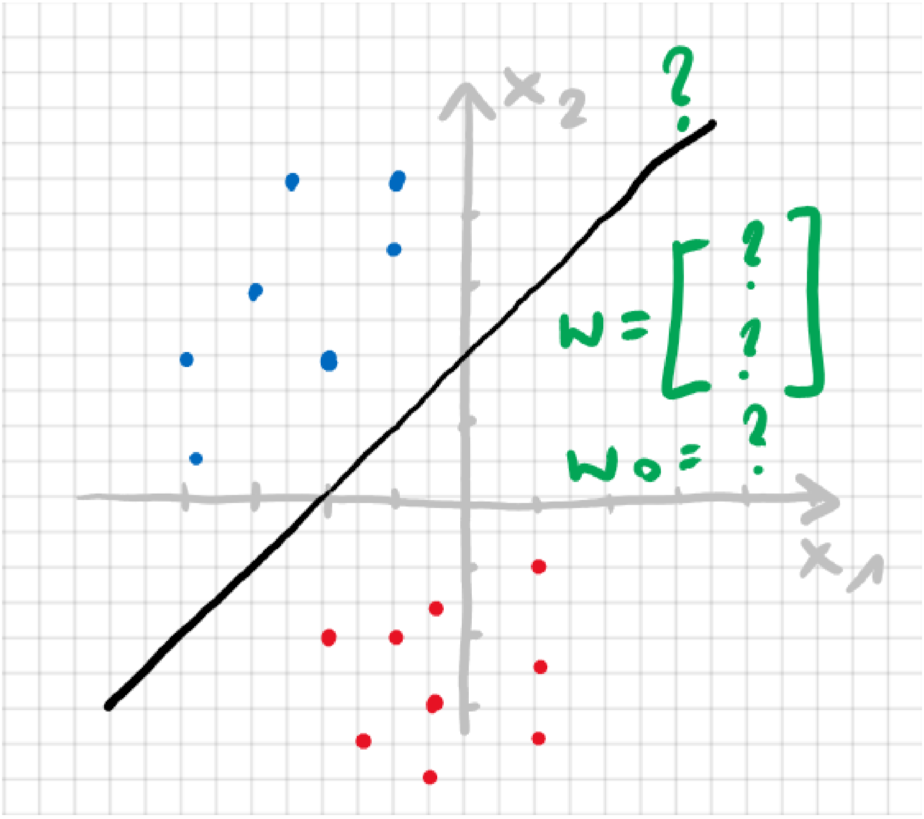
Klasyfikator liniowy

$$\underbrace{w_1 x_1 + w_2 x_2 + w_0 = 0}_{w^T \cdot x + w_0 = 0}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$w_0$$

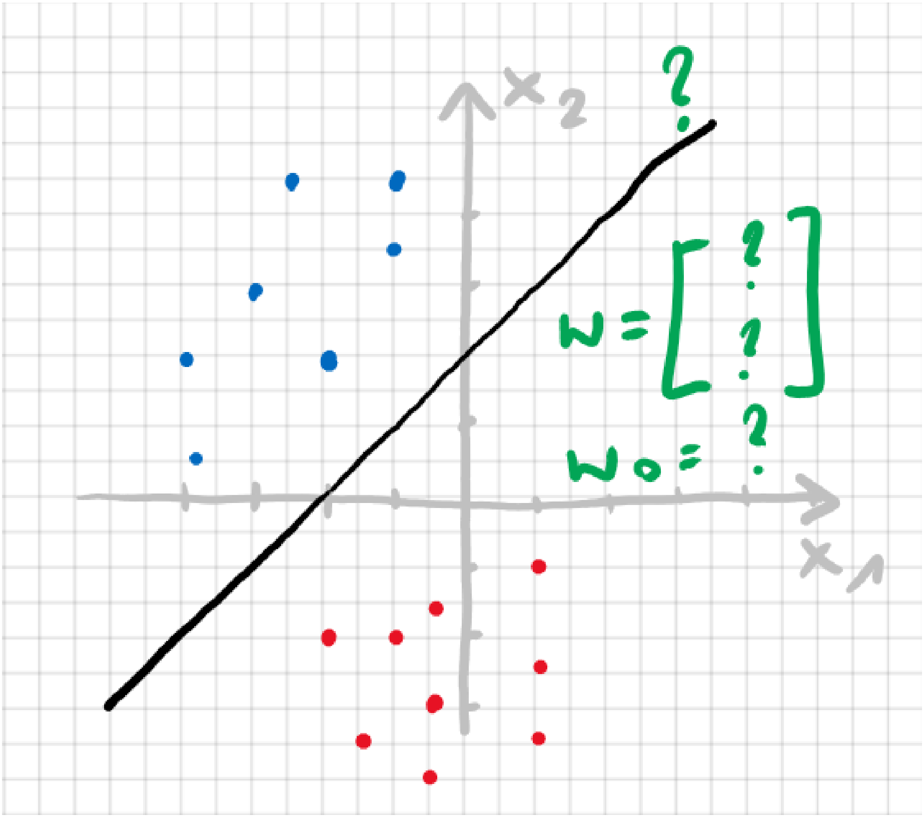


Ogólna postać klasyfikatora liniowego

$$\delta(x) = w^T \cdot x + w_0$$

$$d(x) = \mathrm{sign}(\delta(x)) = \begin{cases} 1 & \Leftrightarrow \delta(x) > 0 \\ -1 & \Leftrightarrow \delta(x) \leq 0 \end{cases}$$

$$\underbrace{w_1 x_1 + w_2 x_2 + w_0 = 0}_{w^T \cdot x + w_0 = 0}$$



$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

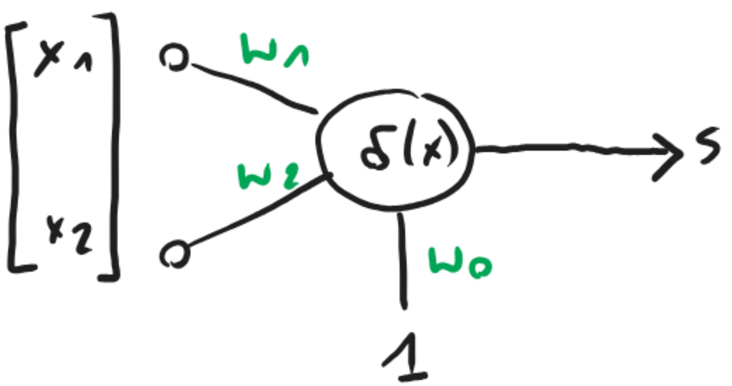
$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$w_0$$

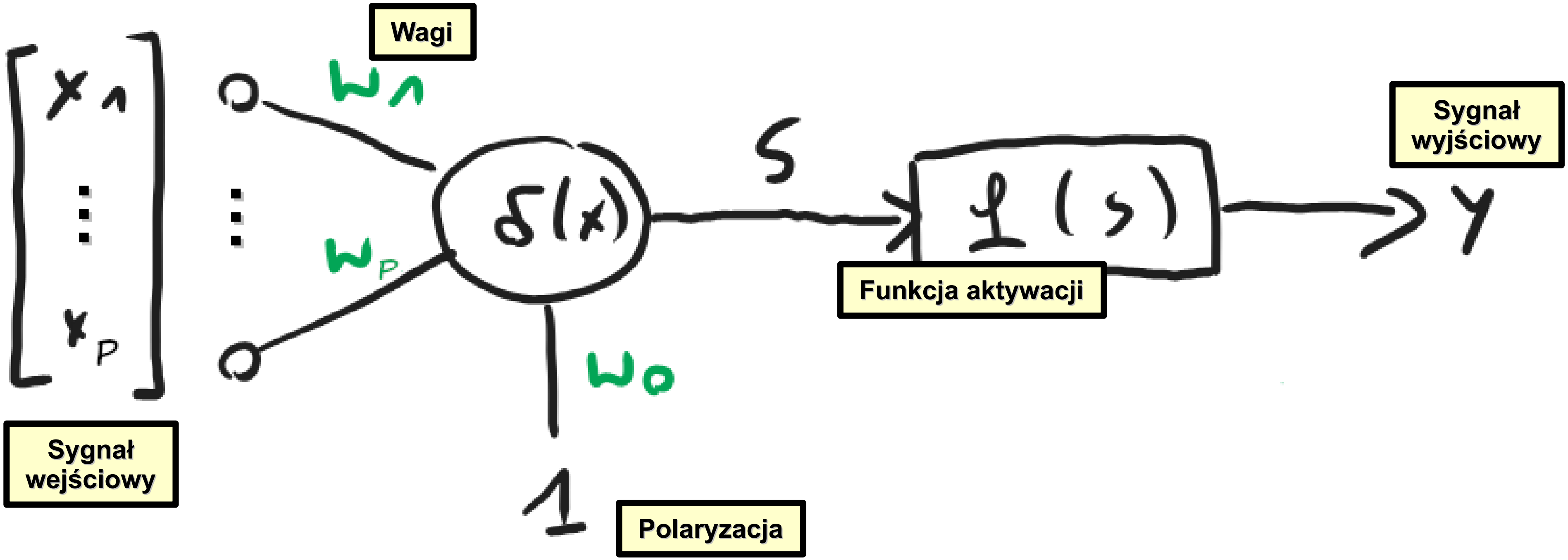
Ogólna postać klasyfikatora liniowego

$$\delta(x) = w^T \cdot x + w_0$$

$$d(x) = \mathrm{sign}(\delta(x)) = \begin{cases} 1 & \Leftrightarrow \delta(x) > 0 \\ -1 & \Leftrightarrow \delta(x) < 0 \end{cases}$$



Sieci typu MLP: sztuczny neuron (budowa)



$$y = f(s) = f(\delta(x)) = f(w^T \cdot x + w_0)$$

Sieci typu MLP: sztuczny neuron (ogólny schemat uczenia)

Funkcja celu

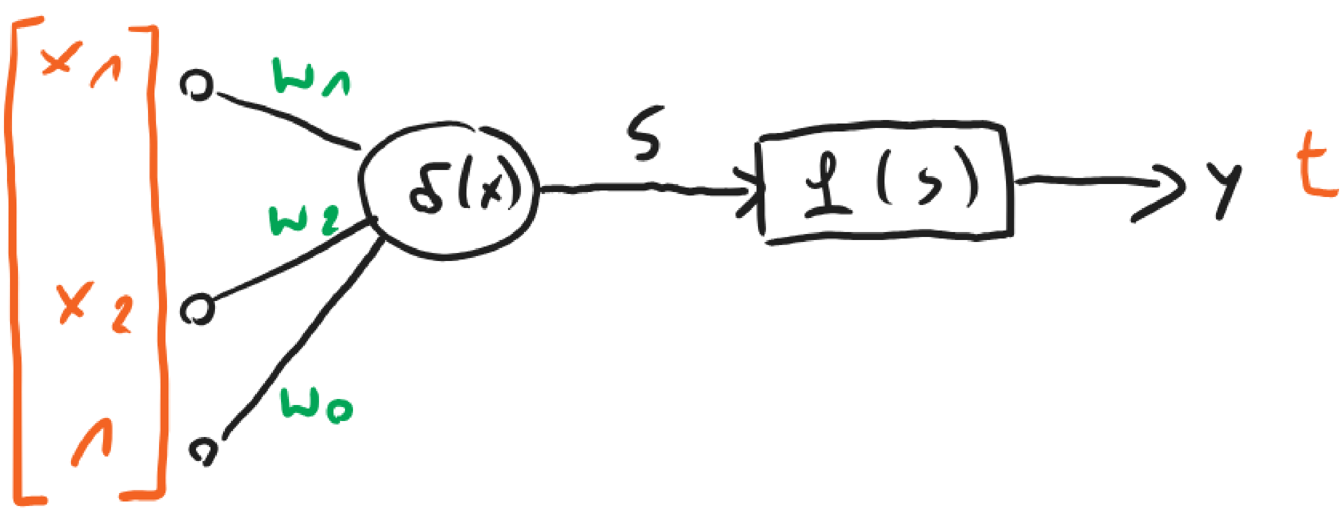
$$E = \frac{1}{2}(y-t)^2 = \frac{1}{2}(f(s)-t)^2$$

Przykład uczący

$$\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} ; t \right\}$$

Sposób aktualizacji wag

$$w_i^{(k+1)} = w_i^{(k)} - \eta\,(y-t)\cdot x \cdot \frac{\partial f(s)}{\partial s}$$



Sieci typu MLP: neuron liniowy

Funkcja celu

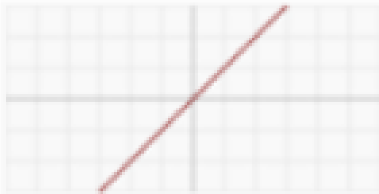
$$E = \frac{1}{2}(y-t)^2 = \frac{1}{2}(f(s)-t)^2$$

Przykład uczący

$$\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} ; t \right\}$$

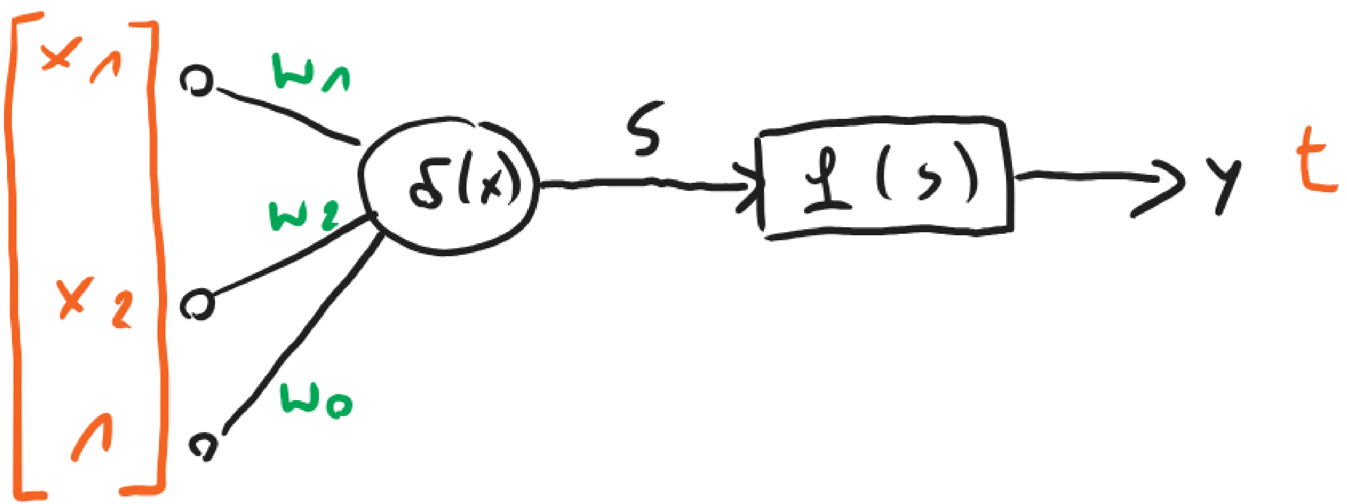
Liniowa funkcja aktywacji

$$f(s) = s$$



Sposób aktualizacji wag

$$w_i^{(k+1)} = w_i^{(k)} - \eta (y-t) \cdot x$$



Sieci typu MLP: neuron nieliniowy (sigmoidalny bipolarny)

Funkcja celu

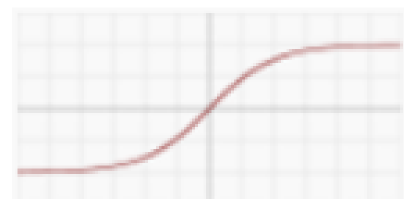
$$E = \frac{1}{2}(y-t)^2 = \frac{1}{2}(f(s)-t)^2$$

Przykład uczący

$$\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} ; t \right\}$$

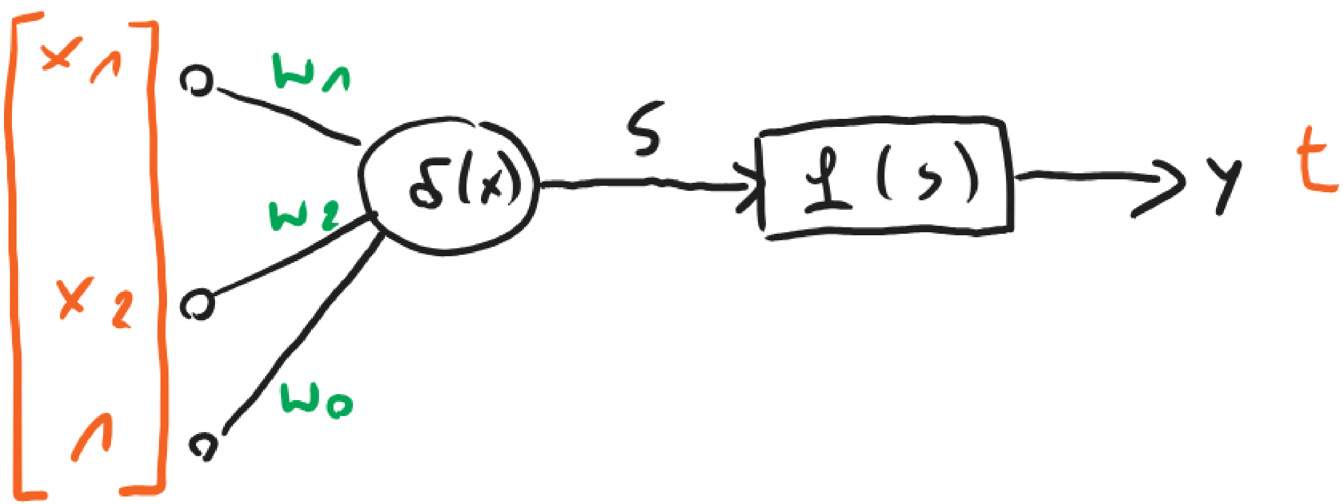
Nieliniowa funkcja aktywacji

$$f(s) = \frac{2}{1+e^{-2s}} - 1$$

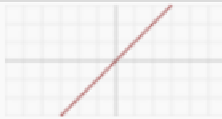
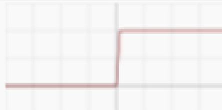
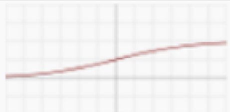
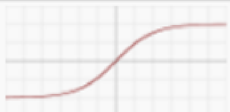
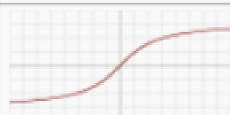
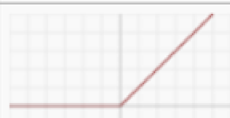
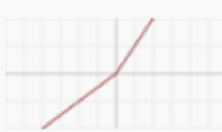
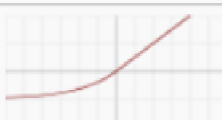
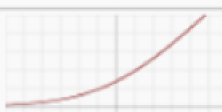


Sposób aktualizacji wag

$$w_i^{(k+1)} = w_i^{(k)} - \eta (y-t) \cdot x \cdot (1 - f^2(s))$$



Sieci typu MLP: neuron nieliniowy (funkcje aktywacji)

Name	Plot	Equation	Derivative
Identity		$f(x) = x$	$f'(x) = 1$
Binary step		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ ? & \text{for } x = 0 \end{cases}$
Logistic (a.k.a Soft step)		$f(x) = \frac{1}{1 + e^{-x}}$	$f'(x) = f(x)(1 - f(x))$
TanH		$f(x) = \tanh(x) = \frac{2}{1 + e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2 + 1}$
Rectified Linear Unit (ReLU)		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Parameteric Rectified Linear Unit (PReLU)[2]		$f(x) = \begin{cases} \alpha x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Exponential Linear Unit (ELU)[3]		$f(x) = \begin{cases} \alpha(e^x - 1) & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} f(x) + \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
SoftPlus		$f(x) = \log_e(1 + e^x)$	$f'(x) = \frac{1}{1 + e^{-x}}$

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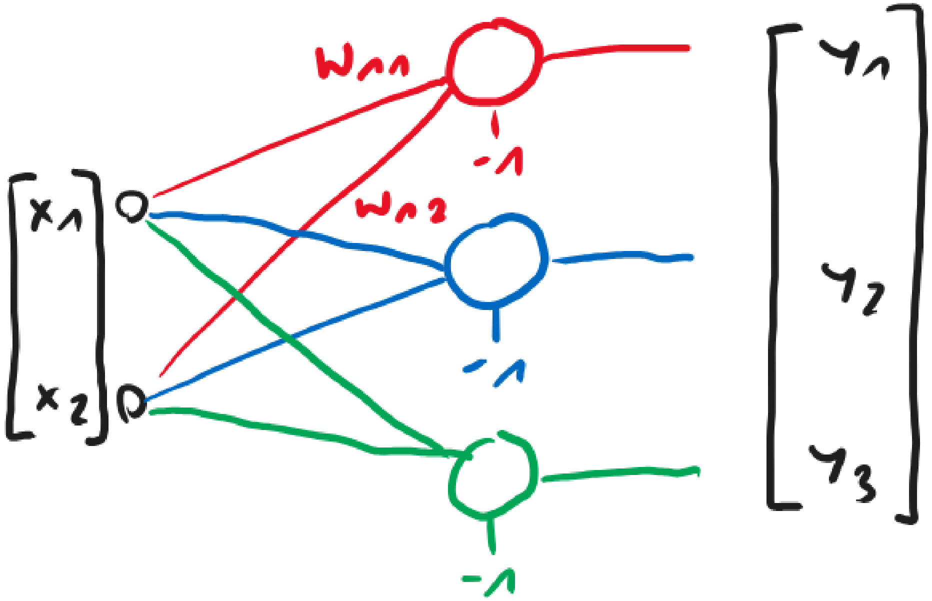
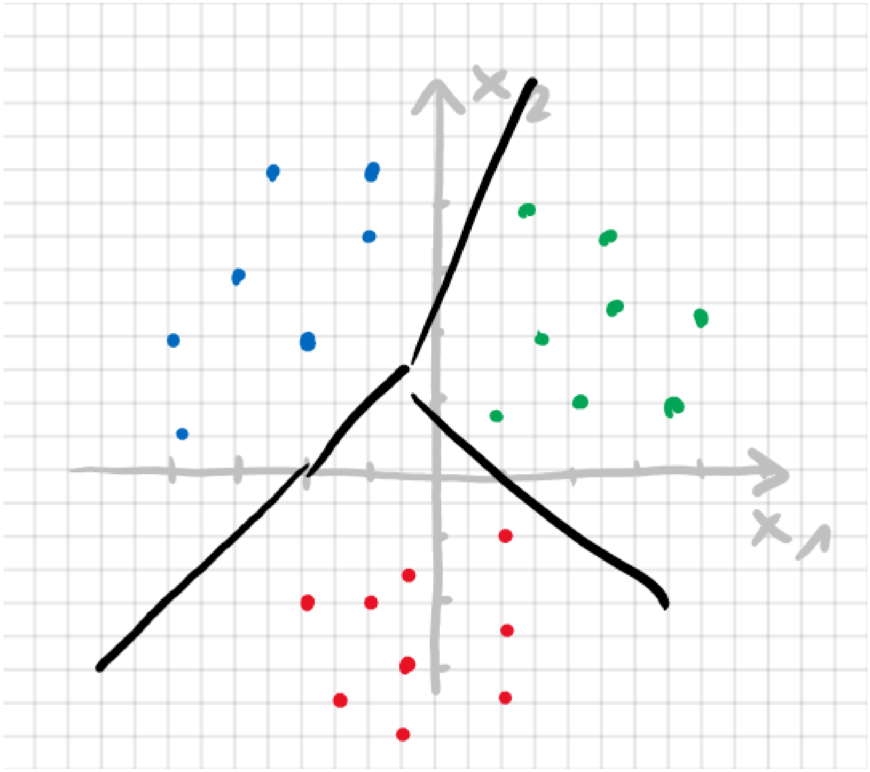
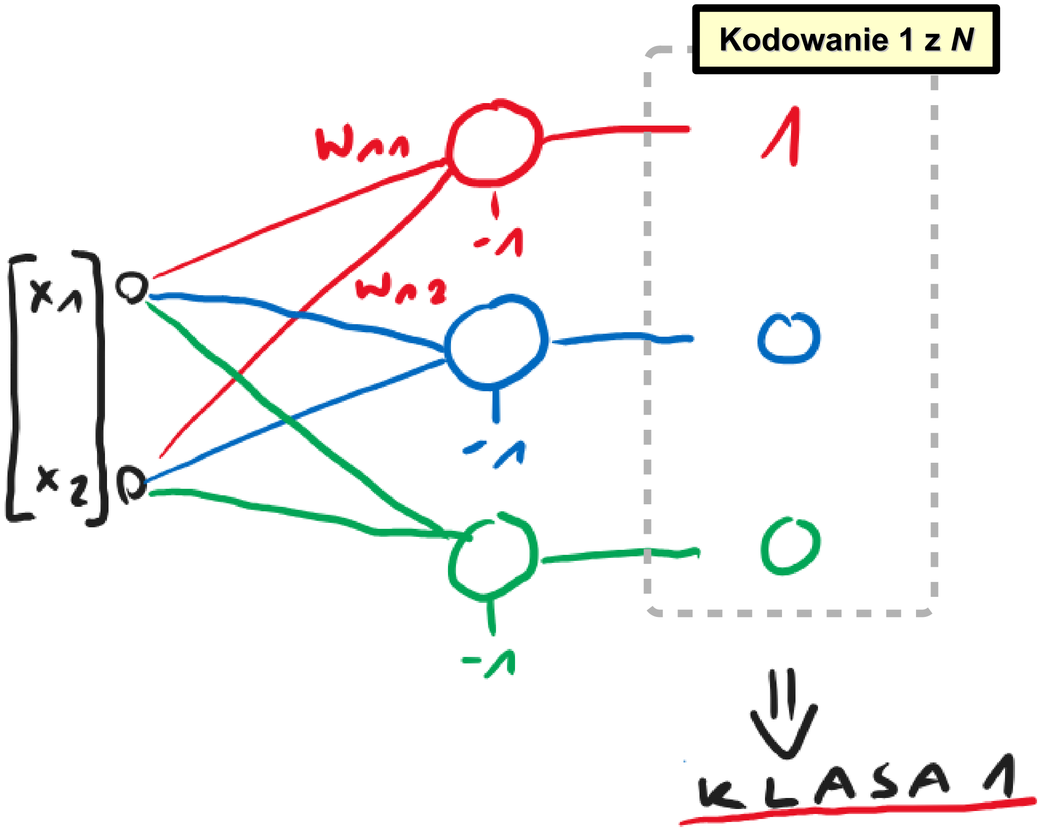
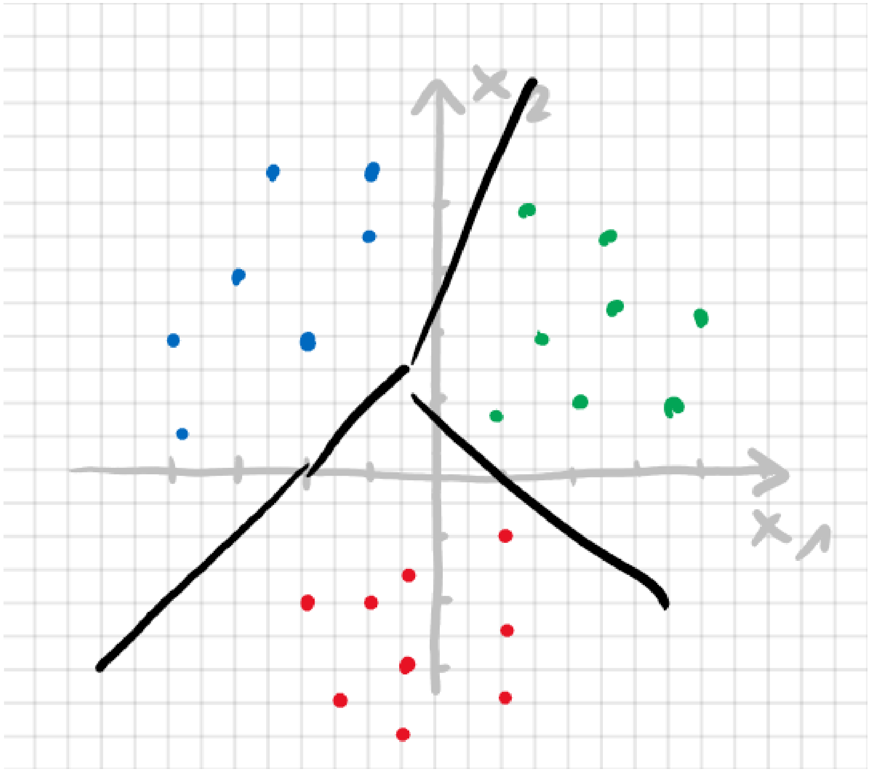
Sieci typu MLP: warstwa neuronów



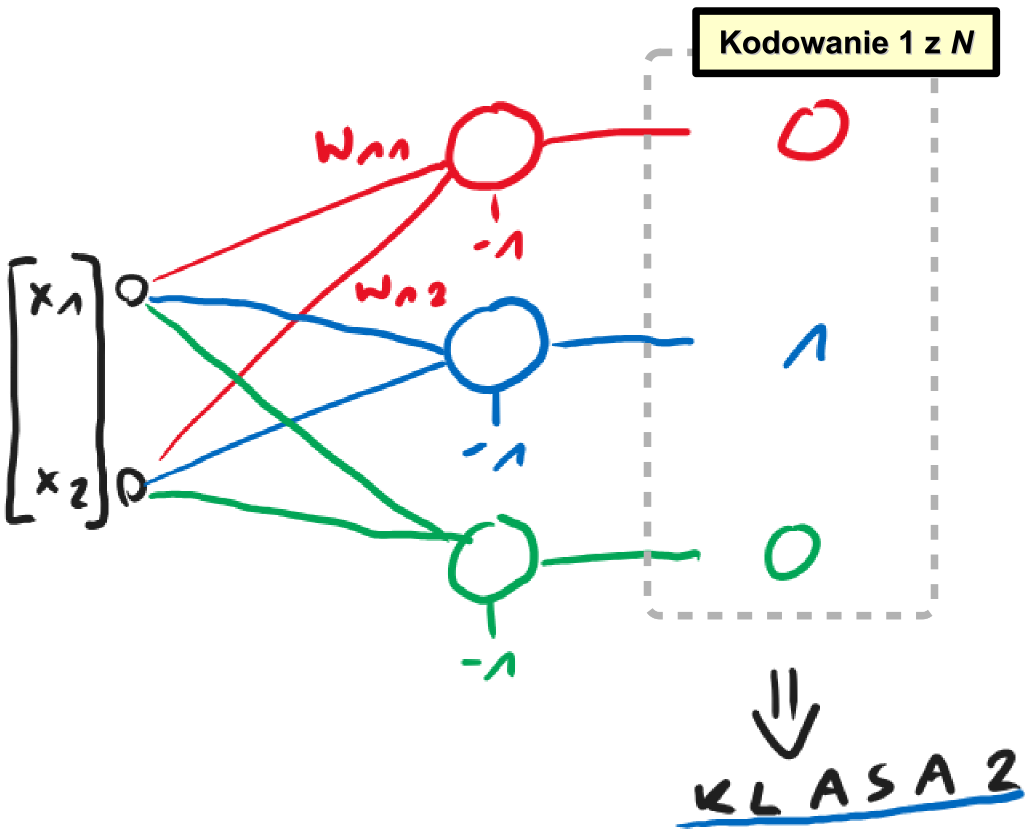


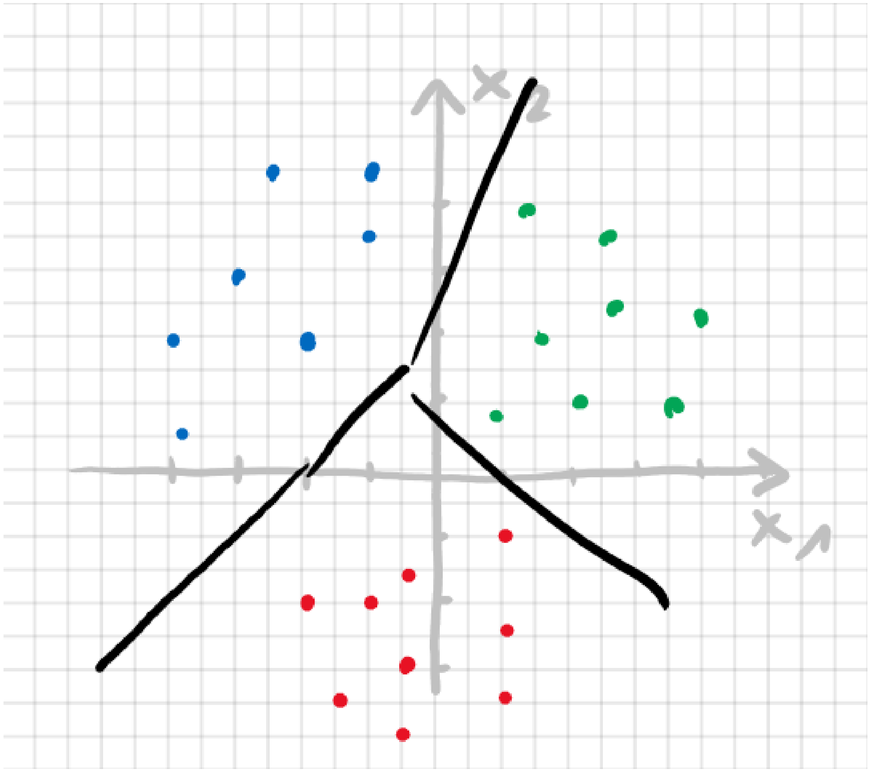
Sieci typu MLP: warstwa neuronów



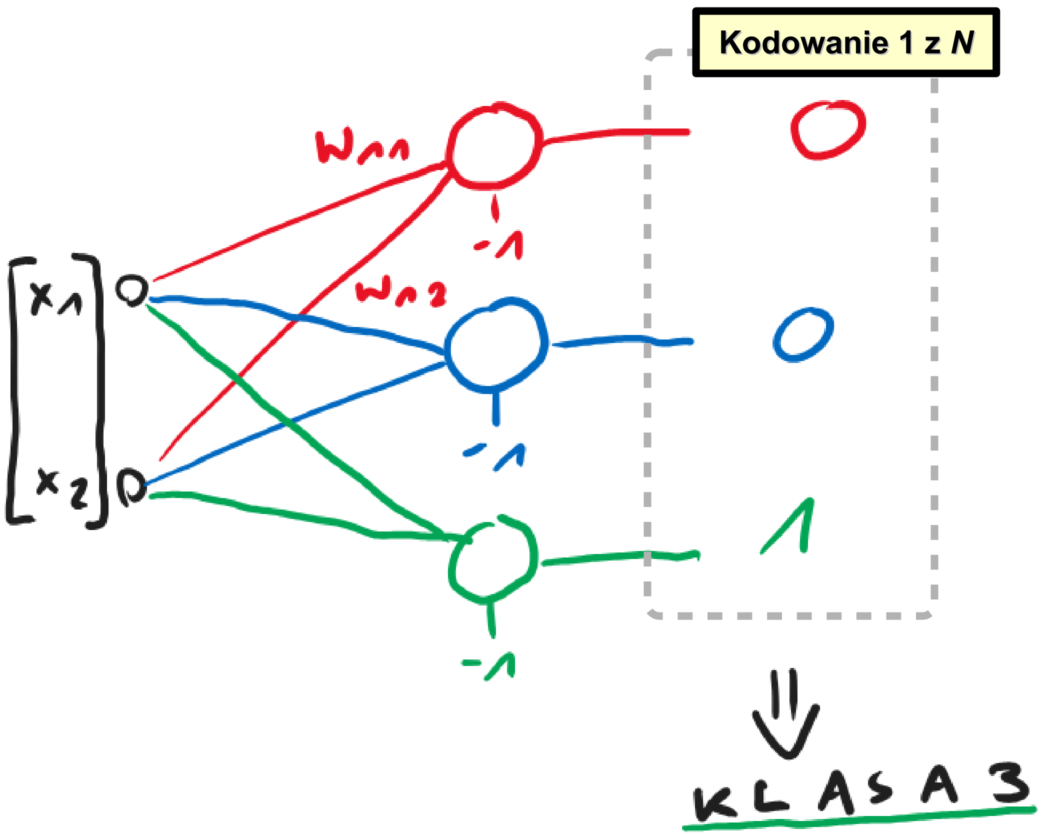


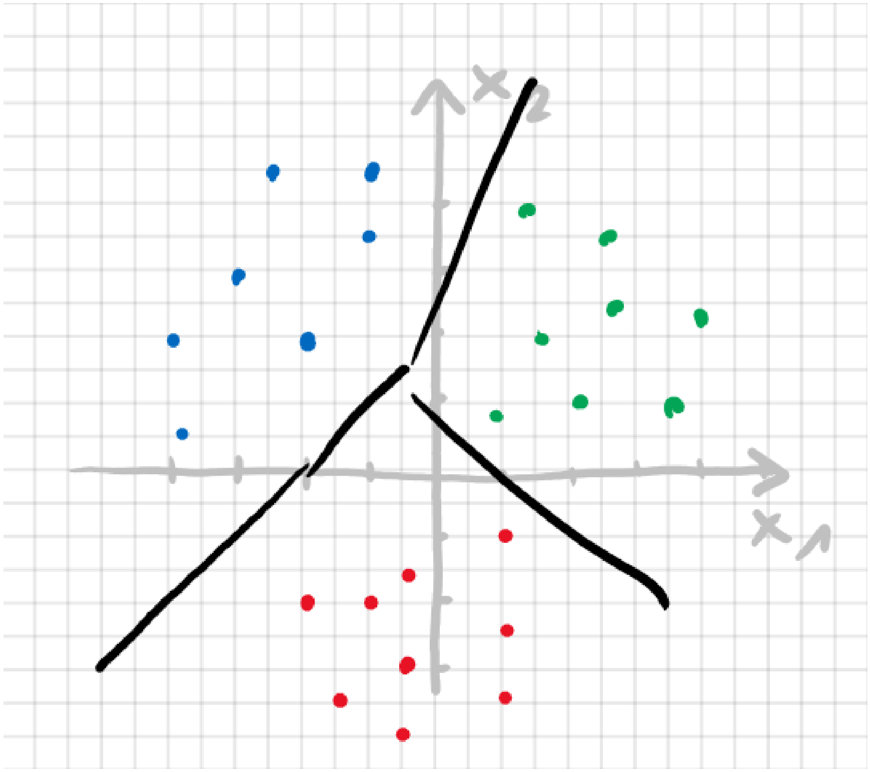
Sieci typu MLP: warstwa neuronów



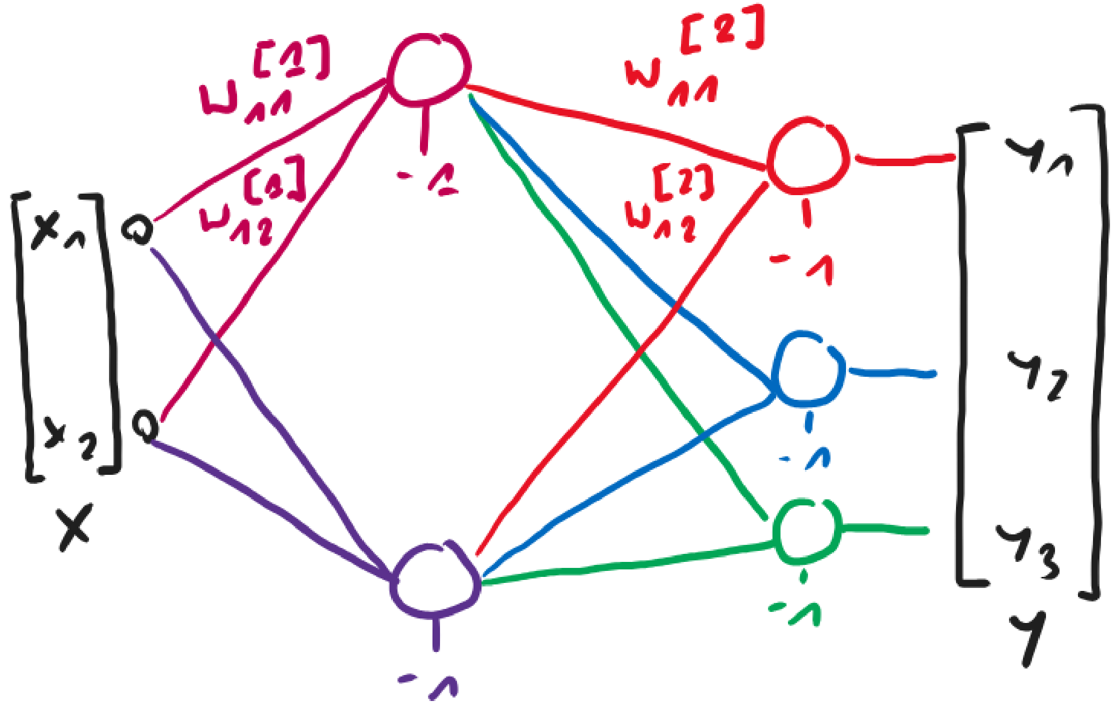


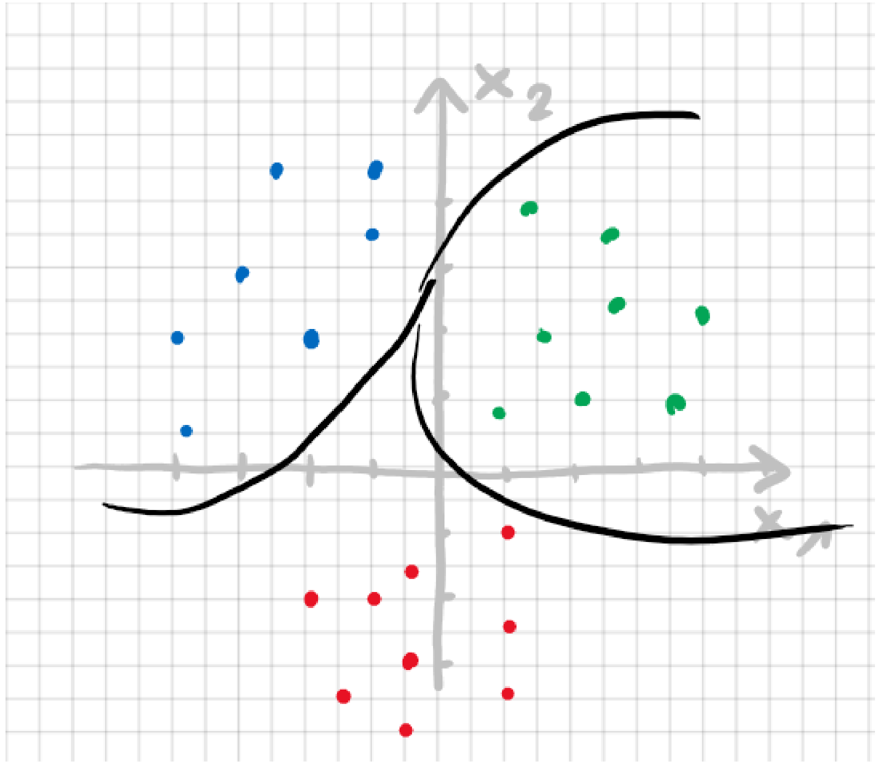
Sieci typu MLP: warstwa neuronów



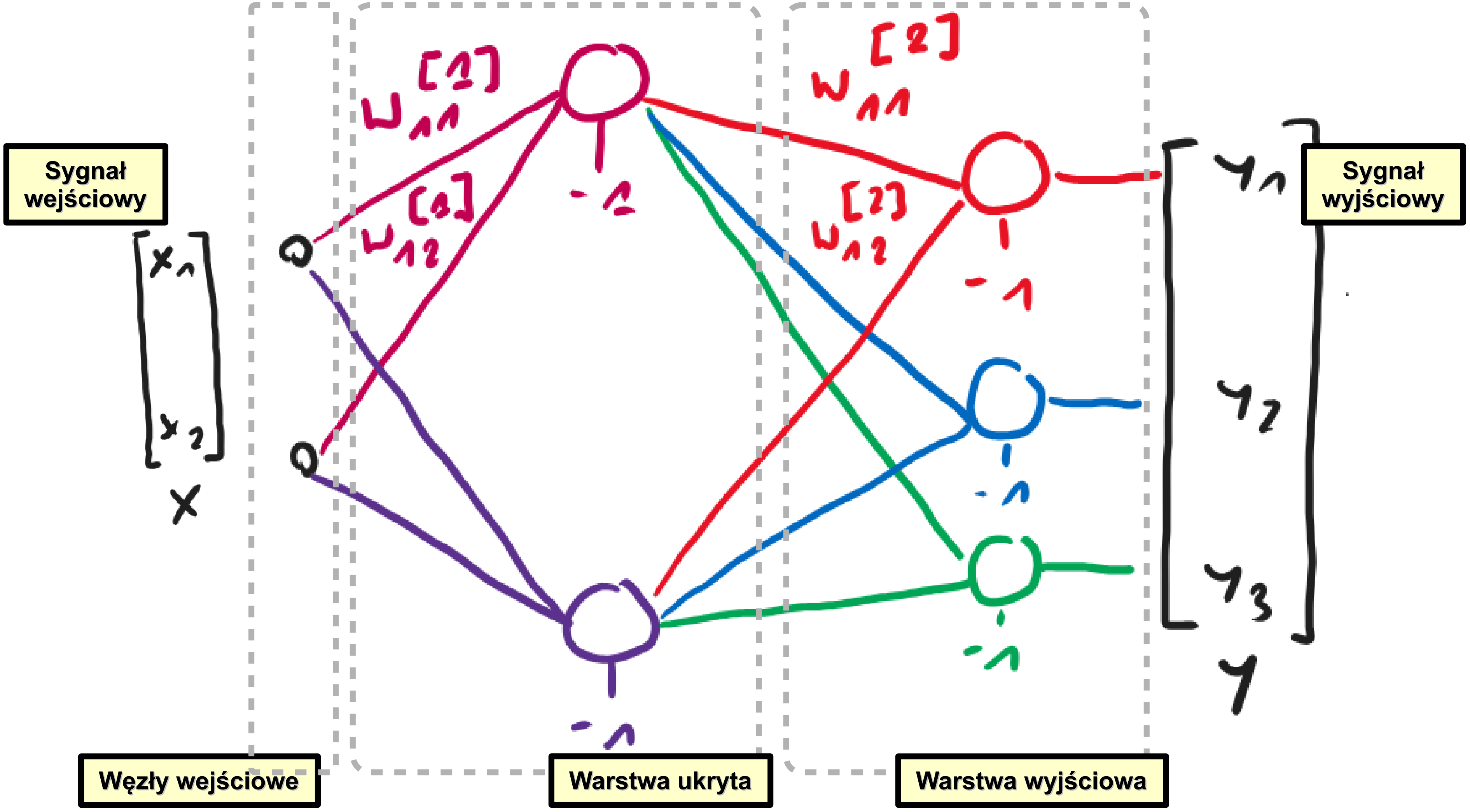


Sieci typu MLP: sieć wielowarstwowa

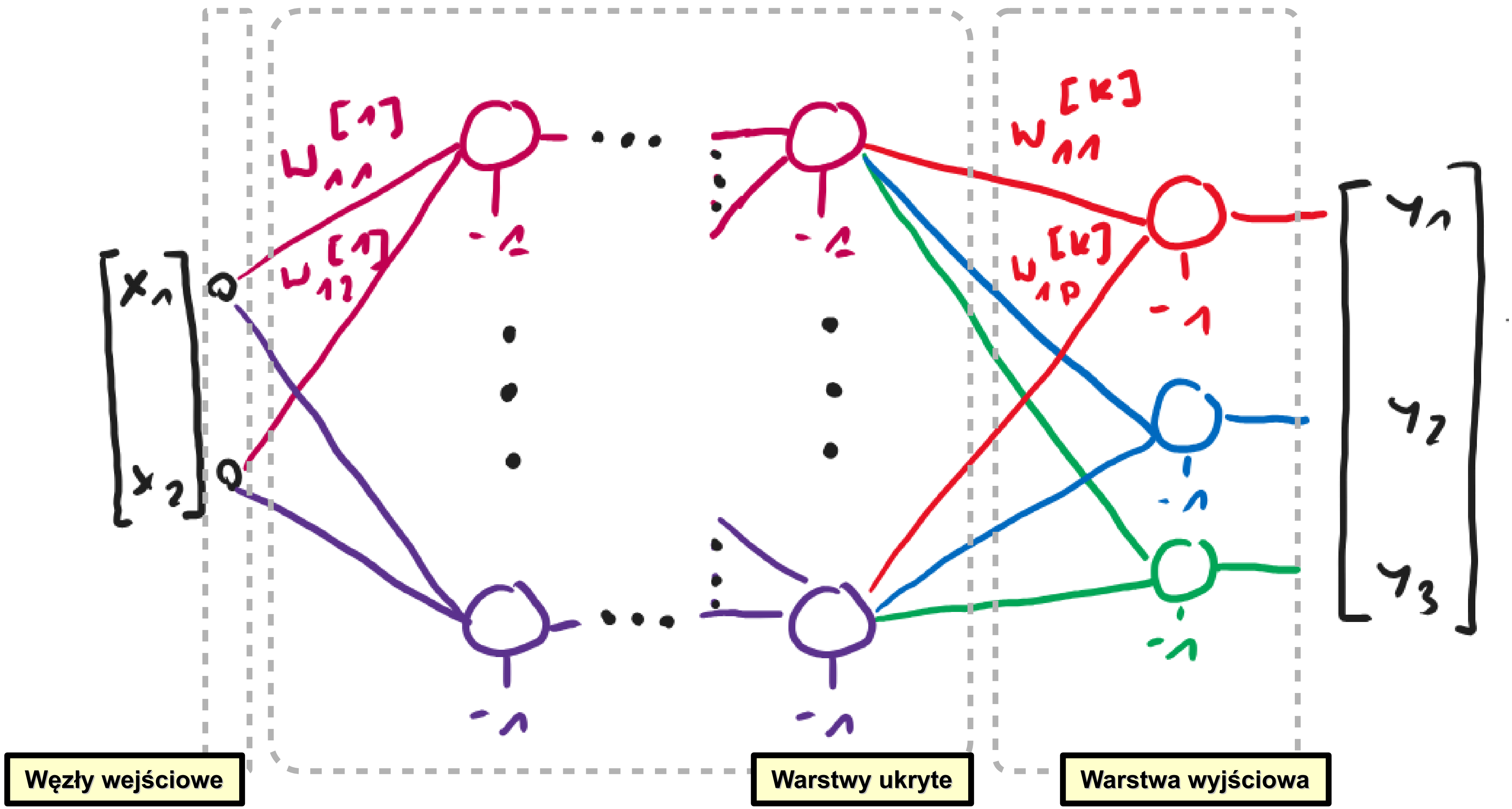




Sieci typu MLP: sieć wielowarstwowa



Sieci typu MLP: sieć wielowarstwowa



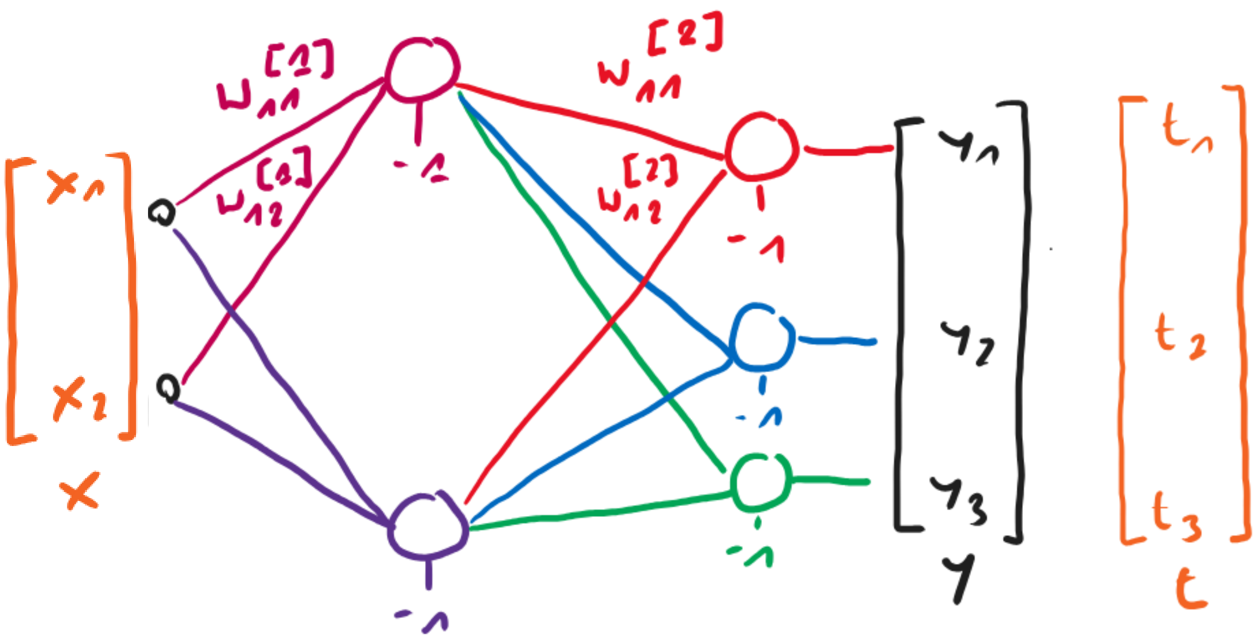
Sieci typu MLP: sieć wielowarstwowa

$$w_{ij}(k+1) = w_{ij}(k) - \eta \cdot (y-t) \cdot \frac{\partial f(s)}{\partial s} \cdot x =$$

$$= w_{ij}(k) - \eta \cdot e \cdot \frac{\partial f(s)}{\partial s} \cdot x = w_{ij}(k) - \eta \cdot d \cdot x$$

$$e = (y-t)$$

$$d = e \cdot \frac{\partial f(s)}{\partial s}$$



Sieci typu MLP: sieć wielowarstwowa

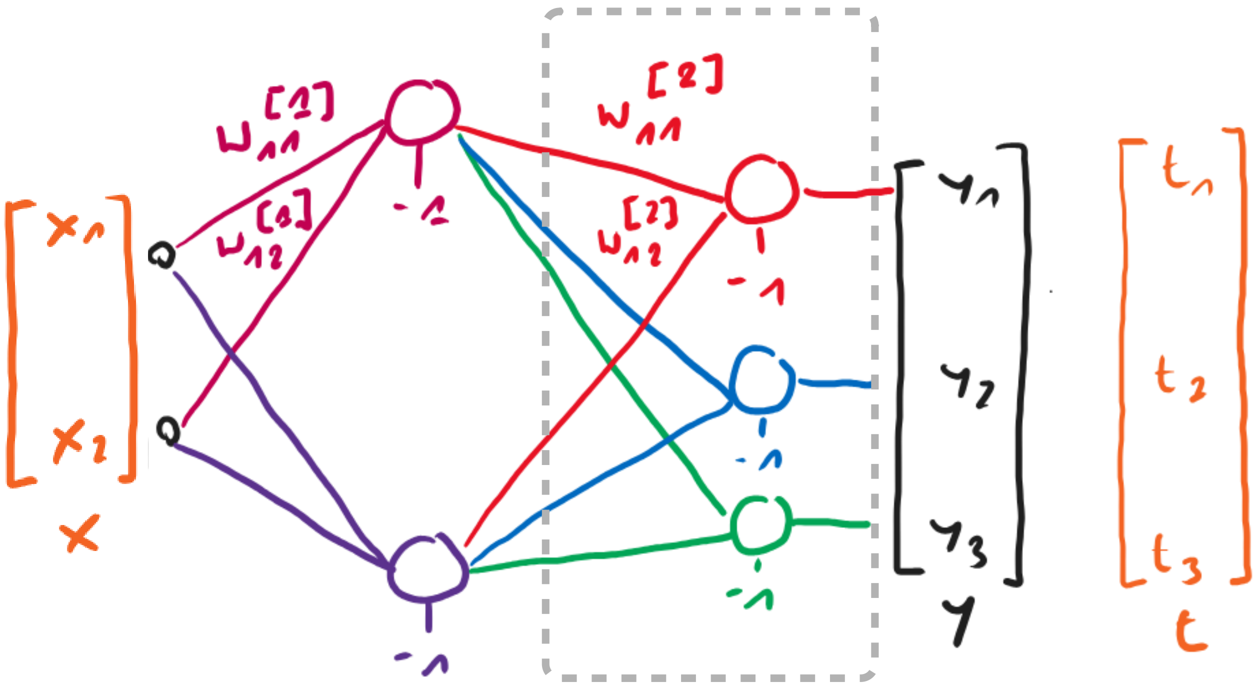
Sposób aktualizacji wag warstwy wyjściowej

$$w_{ij}^{[2]}(k+1) = w_{ij}^{[2]}(k) - \eta\, d^{[2]} \cdot x^{[2]}$$

$$x^{[2]} = y^{[1]}$$

$$d^{[2]} = e^{[2]} \cdot \frac{\partial f(s^{[2]})}{\partial s^{[2]}}$$

$$e^{[2]} = (y - t)$$



Sieci typu MLP: sieć wielowarstwowa

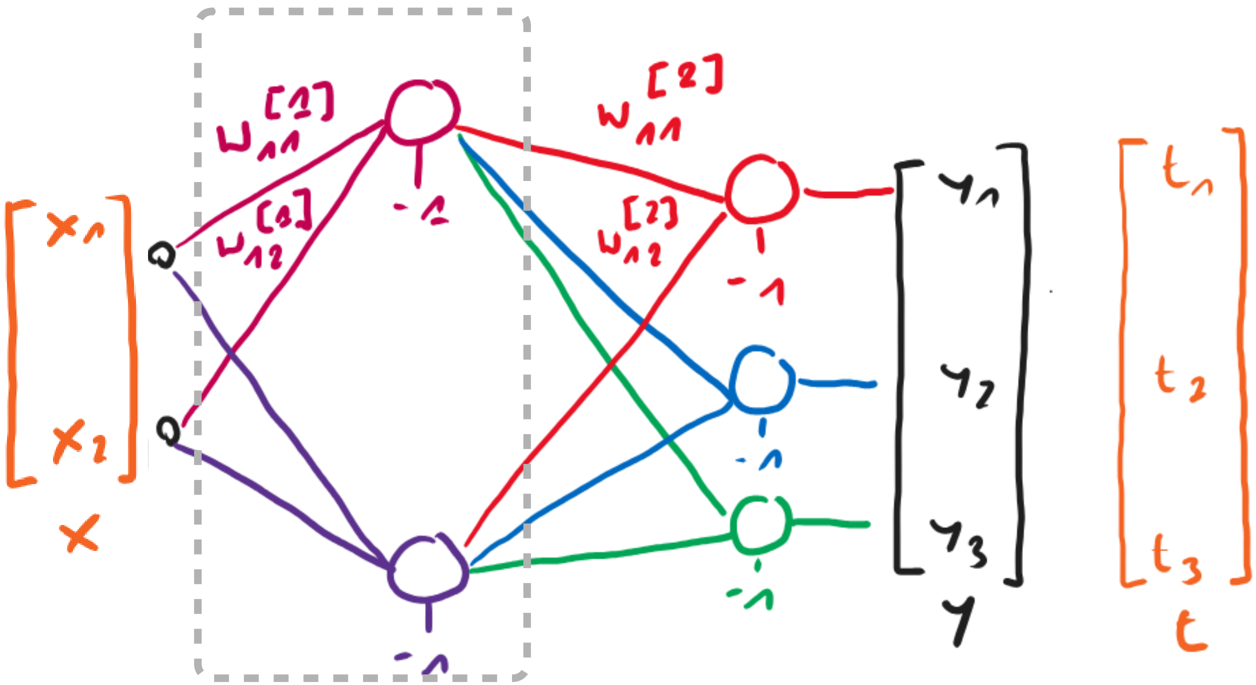
Sposób aktualizacji wag warstwy ukrytej

$$w_{ij}^{[1]}(k+1) = w_{ij}^{[1]}(k) - \eta\, d^{[1]} \cdot x^{[1]}$$

$$x^{[1]} = x$$

$$d^{[1]} = e^{[1]} \cdot \frac{\partial f(s^{[1]})}{\partial s^{[1]}}$$

$$e^{[1]} = ?$$



Sieci typu MLP: sieć wielowarstwowa

Sposób aktualizacji wag warstwy ukrytej

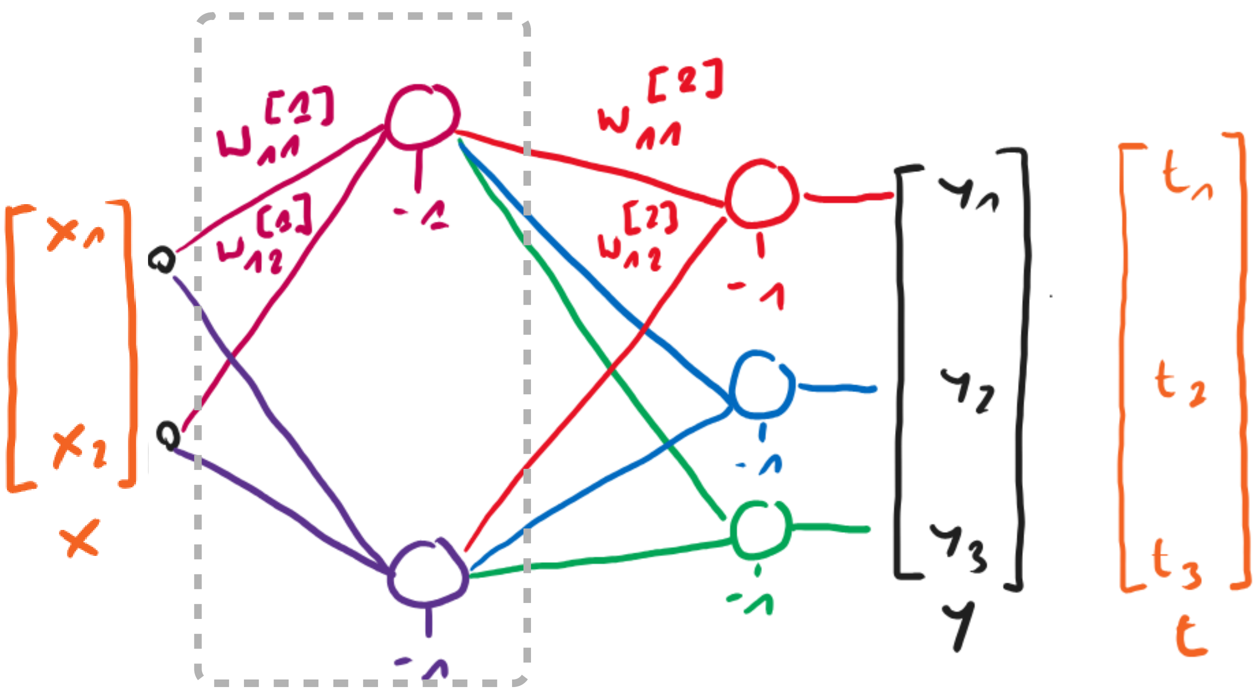
$$w_{ij}^{[1]}(k+1) = w_{ij}^{[1]}(k) - \eta\, d^{[1]} \cdot x^{[1]}$$

$$x^{[1]} = x$$

$$d^{[1]} = e^{[1]} \cdot \frac{\partial f(s^{[1]})}{\partial s^{[1]}}$$

$$e^{[1]} = \sum_{j=1}^{N^{[2]}} w_{1j}^{[2]} \cdot d_j^{[2]}$$

Wsteczna propagacja błędu



Uczenie maszynowe w bioinformatyce

Wykład 10: sztuczne sieci neuronowe typu WTA/WTM

Tymon Rubel

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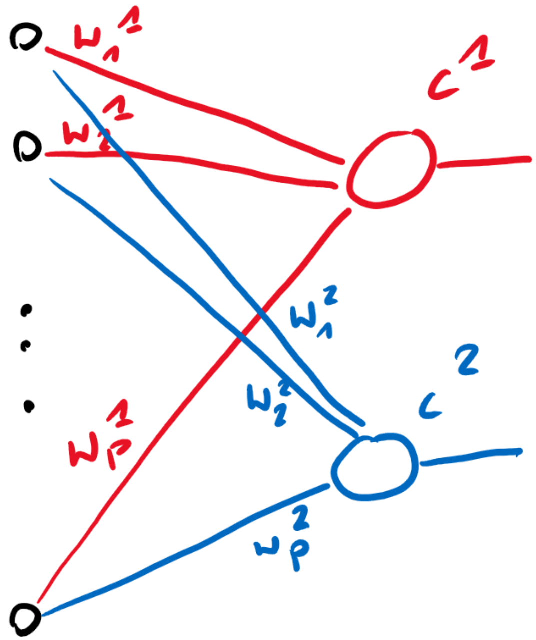
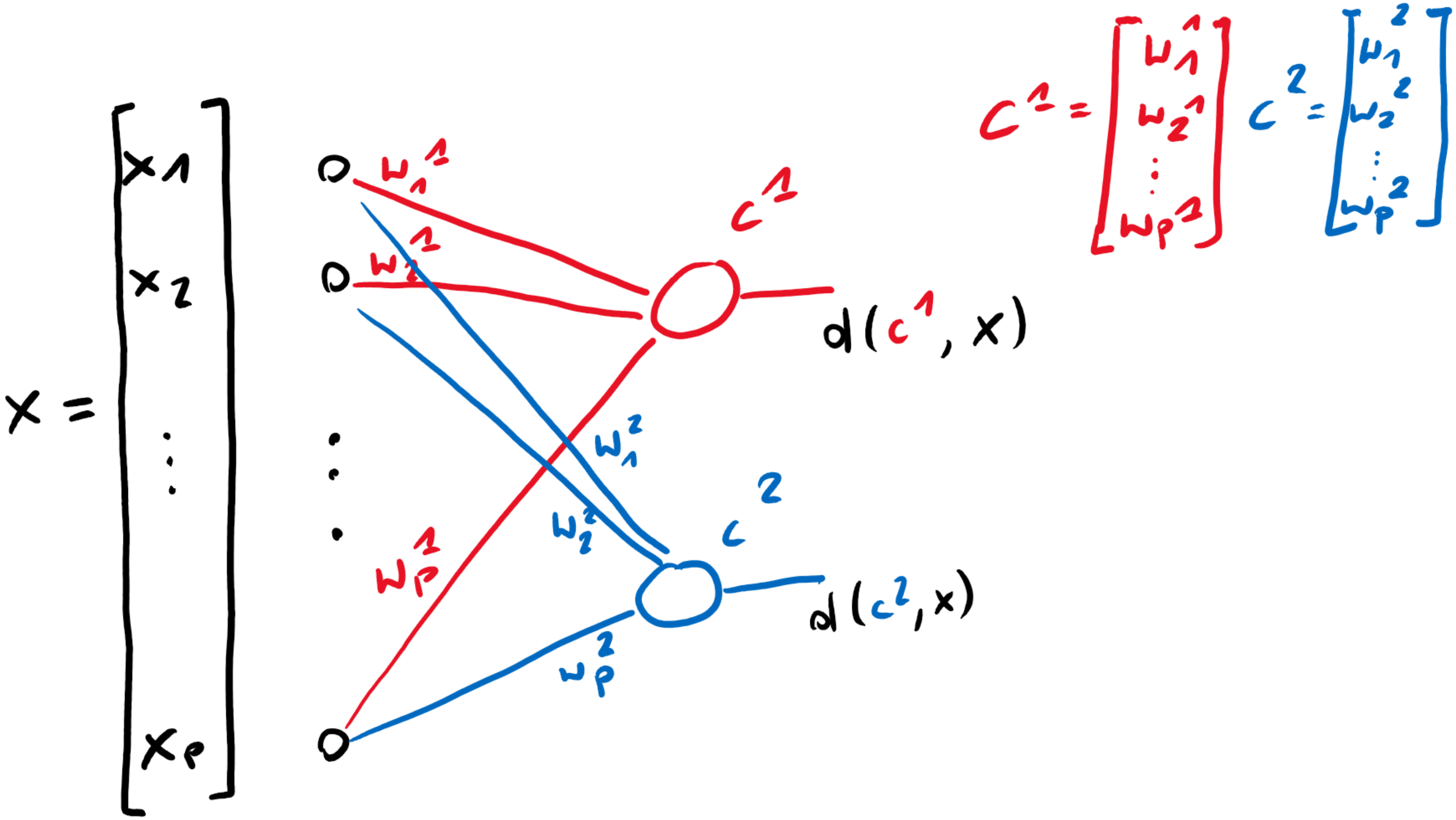
Instytut Radioelektroniki i Technik Multimedialnych PW

Sieci typu WTA/WTM

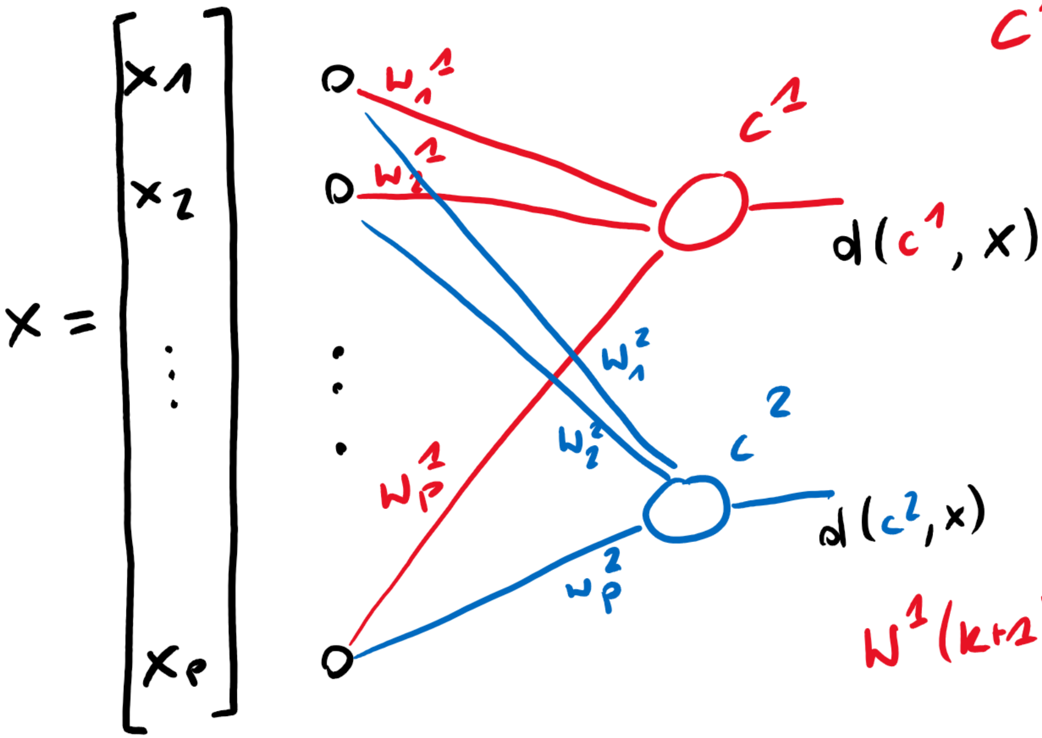


$$C^1 = \begin{bmatrix} w_1^1 \\ w_2^1 \\ \vdots \\ w_p^1 \end{bmatrix} \quad c^2 = \begin{bmatrix} w_1^2 \\ w_2^2 \\ \vdots \\ w_p^2 \end{bmatrix}$$

Sieci typu WTA/WTM



Sieci typu WTA/WTM



$$c^1 = \begin{bmatrix} w_1^1 \\ w_2^1 \\ \vdots \\ w_p^1 \end{bmatrix} \quad c^2 = \begin{bmatrix} w_1^2 \\ w_2^2 \\ \vdots \\ w_p^2 \end{bmatrix}$$

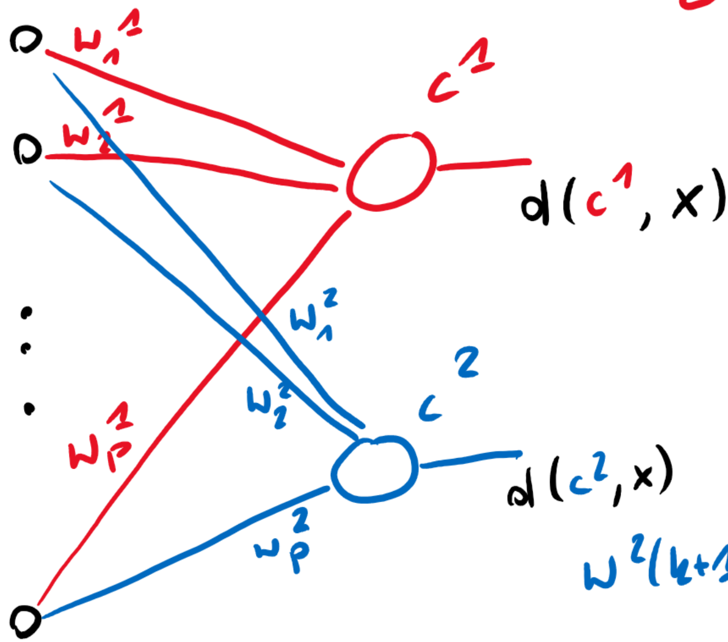
$$d(c^1, x) < d(c^2, x)$$

$$\Downarrow$$

$$w^1(k+1) = \eta \cdot x + (1-\eta) \cdot w^1(k)$$

Sieci typu WTA/WTM

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix}$$



$$c^1 = \begin{bmatrix} w_1^1 \\ w_2^1 \\ \vdots \\ w_p^1 \end{bmatrix} \quad c^2 = \begin{bmatrix} w_1^2 \\ w_2^2 \\ \vdots \\ w_p^2 \end{bmatrix}$$

$$d(c^1, x) > d(c^2, x)$$

$$\Downarrow$$

$$w^2(k+1) = \eta \cdot x + (1-\eta) \cdot w^2(k)$$