# Karta wzorów – kolokwium 1

## Tożsamości trygonometryczne

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x, \quad \sin 2x = 2\sin x \cdot \cos x$$

$$\sin(x \pm y) = \sin x \cdot \cos y \pm \cos x \cdot \sin y, \quad \cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y$$

$$\cos^3 x = \frac{1}{4}(\cos 3x + 3\cos x), \quad \sin^3 x = \frac{1}{4}(-\sin 3x + 3\sin x)$$

$$\cos x \cdot \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right], \quad \sin x \cdot \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\sin x \cdot \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right], \quad \cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right), \quad \sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$

## Podstawowe sygnały

$$r(t) = t \cdot 1(t), \quad \frac{d}{dt}1(t) = \delta(t), \quad x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0), \quad \delta(at) = \frac{1}{|a|}\delta(t)$$

$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau, \quad R_x(\tau) = \int_{-\infty}^{\infty} x(t) x^*(t-\tau) dt$$

## Szereg Fouriera

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \varphi_n), \; x(t) = A_0 + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right],$$

$$a_n = A_n \cos(\varphi_n), \quad b_n = -A_n \sin(\varphi_n), \quad A_n = \sqrt{a_n^2 + b_n^2}$$

$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$, $c_n = c_{-n}^*$ dla sygnałów $x(t)$ o wartościach rzeczywistych

$$A_0 = c_0, \quad A_n = 2|c_n|, \quad a_n = c_n + c_{-n} = c_n + c_n^* = 2\operatorname{Re}(c_n), \quad b_n = j(c_n - c_{-n}) = j(c_n - c_n^*) = -2\operatorname{Im}(c_n)$$

$P = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} |x(t)|^2 \, dt = A_0^2 + \sum_{n=1}^{\infty}\frac{A_n^2}{2} = \sum_{n=-\infty}^{\infty}|c_n|^2$ - moc sygnału okresowego

## Przekształcenie Fouriera

$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega = \int_{-\infty}^{\infty} |X(f)|^2 \, df$ - energia sygnału $x(t)$

Widmo sygnału okresowego $x(t) = \sum_{n=-\infty}^{\infty} z(t - nT_0)$: $\quad X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_0)$,

$$c_n = \frac{1}{T_0} Z(n\omega_0), \quad z(t) \overset{F}{\leftrightarrow} Z(\omega)$$