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Report No. 1, 2019

On the Virtual Energy Aureole of Spherical Bodies

by

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Warsaw, May 2019

Glossary of terms

MKS System of Units
M [kg] – baryonic mass of the sphere
R_0 [m] – Radius of the sphere
ρ_V [kg/m^3] – Baryonic mass density = $3M/\left(4\pi R_0^3\right)$
Vmd [kg/m^3] – virtual mass density of a energetic medium
ρ_S [kg/m^2] – equivalent surface mass density= $M/\left(4\pi R_0^2\right)$
End_g – energy density of the gravitational field of the sphere
End_{QV} [J/m^3] or [N/m^2] or [kg/(ms^2)] – energy density of the aureola surrounding a sphere
G[m^3/(kgs^2)] =6,67384×10^{-11} – the gravitational constant
$\gamma=1/(4\pi G)$ – the reciprocal gravitational constant
c – the speed of light in vacuum free of any energy fields

Abstract: By investigating the gravity fields surrounding spherical bodies of constant mass density, the author discovered that they are surrounded not only by the negative energy density of the gravitational field but also by another energy density field which was named virtual energy density and a connected virtual mass density. The paper is illustrated with data for the solar system, a neutron star and a black hole. The author suggests that the ideas about the origin of gravity presented in his earlier papers are validated again in this paper.

Introduction

The knowledge of the properties of the quantum vacuum (QV) is crucial for understanding and description of the properties of the Universe. Since the famous paper of Casimir [1] we know that the vacuum is a highly energetic medium. In this paper, we study the properties of vacuum around a sphere of radius R_0 and mass M filled with a uniform mass density $\rho_V=3M/\left(4\pi R_0^3\right)\ \left[\text{kg/m}^3\right]$. The gravitational field of this sphere g(r) is a circular symmetric function of the radius r (see Appendix 1, Eq.(A1)). Starting with the Einstein's formula defining the dependence of the speed of light on gravity, we introduced a new notion of ***virtual energy density*** which surrounds bodies with a baryonic mass density. It will be shown that for a $_{V/6.}$body with radius R_0, i.e., a mass density $\rho_V=3M/\left(4\pi R_0^3\right)$, the peak value of the virtual mass density at the surface of the body equals exactly $\rho_V/6$.

The gravity lowers the energy density of the Quantum Vacuum

The space independent energy density of the QV derived from Max Planck formulae for zero absolute temperature $T=0$ (Max Planck half quanta 0.5hf) has a frequency domain spectrum

$$\rho(f)=\frac{4\pi hf^3}{c^3}\ \left[\text{Js/m}^3\right] \tag{1}$$

and the total energy intensity in a frequency band from f_1 to f_2 is

$$End_{Planck}=\int_{f_1}^{f_2}\frac{4\pi hf^3}{c^3}df=\frac{\pi h}{c^3}\left(f_2^4-f_1^4\right)\left(\frac{\text{J}}{\text{m}^3}\right) \tag{2}$$

An unsolved problem of the theory of the energy density of QV is the lowering of this energy density by the gravitational field g [m/s^2]. The derivation of the following formula for the energy density of the g-field

$$End_g=-0.5\gamma\left|\vec{g}\right|^2\ \left[\text{J/m}^3\right] \tag{3}$$

is simple, since the cancelling of a gravitational field is possible at the cost of positive energy. Therefore, the negative sign cannot be questioned. If we assume that energy densities should be positive, we should write

$$End_{QV} = End_{Planck} - 0.5\gamma|\vec{g}|^2 \ \left[\mathrm{J/m^3}\right] \tag{4}$$

This formula suggests that the gravitational field ***lowers the energy density*** of QV. This lowering occurs around any mass body which is surrounded by a gravitational field.

The notion of a virtual mass density

In this paper, we apply two kinds of mass densities: The first is the mass density of baryonic matter of bodies of total mass *M* and the second one is a virtual mass density. The baryonic mass density is defined as follows: $\rho_V = M/V \ \left[\mathrm{kg/m^3}\right]$, where *M* is the mass and V the volume. For a sphere with radius R_0, $\rho_V = 3M/\left(4\pi R_0^3\right)$. The notion of a virtual mass density ***Vmd*** [kg/m^3] for a baryonic matter free medium of energy density *End* [J'm^3] is defined using the following form of Einstein's formula $End = Vmd \times c^2 \ \left[\mathrm{J/m^3}\right]$ (see the next derivation).

The derivation of the virtual mass density for a gravitational field

The derivation of *Vmd* starts with Einstein's formula [2] defining the dependence of the speed of light on gravity:

$$c_g = c\left(1 + \frac{\phi}{c^2}\right) = c\left(1 - \frac{|\phi|}{c^2}\right) \tag{5}$$

The function ϕ is called the potential of the gravitational field. We applied the absolute value to indicate that it has a negative sign. For a circular symmetric body, it is a circular symmetric function of the radius *r*. This function has a dimension $\phi(r) \ \left[\mathrm{J/kg}\right]$ or $\left[\mathrm{m}^2/s^2\right]$. The same dimension has the square of the speed of light. The multiplication of these terms by the virtual mass density yields

$$c_g = c\left(1 - \frac{|Vmd||\phi|}{|Vmd|c^2}\right) \tag{6}$$

We applied again the absolute values of *Vmd*. The sign of this function should be determined by next derivations. Note that we got in (5) a quotient of energy densities. In the nominator

$$Vmd \ \left[\mathrm{kg/m^3}\right] \times \phi \ \left[\mathrm{J/kg}\right] \Rightarrow \ \left[\mathrm{J/m^3}\right] \tag{7}$$

The same for the product $Vmd \times c^2$. The formula (2) can be written in the form

$$c_g = c\left(1 - \frac{\textit{Energy density of gravitation}}{\textit{Virtual energy density}}\right) \tag{8}$$

Since absolute value of the energy density of the gravitational field is

$$End_g = 0.5\gamma|\vec{g}|^2 \ \left[\mathrm{J/m^3}\right] \tag{9}$$

the value of the virtual mass density is defined by

$$|Vmd| = \frac{0.5\gamma|\vec{g}|^2}{|\phi|} \ \left[\text{kg/m}^3\right] \tag{10}$$

For a circular symmetric body it is a circular symmetric function. The corresponding circular symmetric virtual energy density is

$$Ved_{QV} = Vmd \times c^2 \ \left[\text{J/m}^3\right] \tag{11}$$

This energy density is forming a local aureole around a sphere.

Experimental validation of the existence of virtual mass density

We derived the notion of virtual mass density starting with Einstein's formula (2). The experimental validation of this formula validates also our derivation. The scientists of NIST have experimentally validated the dependence on gravity the frequency of a laser frequency standard [3]. It depends on gravity in exactly the same form as (5)

$$f_g = f\left(1 - \frac{|\phi|}{c^2}\right) \tag{12}$$

and the fractional forms have the same values

$$\frac{\Delta c}{c} = \frac{\Delta f}{f} = \frac{|\phi|}{c^2} \tag{13}$$

Therefore, experimental validation of (13) validates also (5).

Calculations of the above functions for a spherical bodies representing the solar system, a neutron star and a black body.

The calculations apply for a spherical body of a uniform mass density ρ_V[kg/m^3]. For example in the case of Earth, the surface *g*- field using uniform mass density equals 9.9031 instead of 9.81 [m/s^3]. The calculated data are presented in Tables 1 and 2.These data present numerical evaluation of the formulae of Appendix 1.

Table 1 Data for the solar system

Name	R_0 [m]	Mass [kg]	ρ_V. [kg/m^3]	$g(R_0)$ [m/s^2]	$End_g(R_0)$ [J/m^3]	$Vmd(R_0)$ [kg/m^3]	$End_{Ein}(R_0)$ [J/m^3]
Sun	1.9381..$\times 10^{30}$	1.9381..$\times 10^{30}$	1397.2	272.23	4.3537..$\times 10^{13}$	232.87	2.8928..$\times 10^{18}$
Mercury	2.42726..$\times 10^{6}$	3.3092..$\times 10^{23}$	5505.9	3.7366	8.3246..$\times 10^{9}$	917.6	8.246..$\times 10^{19}$
Venus	6.02276..$\times 10^{6}$	5.3221..$\times 10^{24}$	5332.1	8.9233	4.7888..$\times 10^{10}$	887.23	7.9733..$\times 10^{19}$
Earth	6.347..$\times 10^{6}$	5.9783..$\times 10^{24}$	5841.7	9.9039	5.8478..$\times 10^{10}$	930.28	8.3602..$\times 10^{19}$
Mars	3.3861..$\times 10^{6}$	6.4275..$\times 10^{23}$	3748.7	3.7371	8.32666..$\times 10^{9}$	657.98..	5.9131..$\times 10^{19}$
Jupiter	7.11459$\times 10^{7}$	1.88992..$\times 10^{2}$	1259.4	25.050	3.7411..$\times 10^{11}$	209.916	1.8864..$\times 10^{19}$
Saturn	5.9976..$\times 10^{7}$	5.6891..$\times 10^{26}$	629.53	10.555	6.6421..$\times 10^{10}$	104.922	9.4290..$\times 10^{18}$
Uran	2.5437..$\times 10^{7}$	8.6901..$\times 10^{25}$	1260.4	8.96317	4.78069..$\times 10^{10}$	210.075	1.88789..$\times 10^{19}$
Neptun	2.5435..$\times 10^{7}$	8.6901..$\times 10^{25}$	1260.7	8.9645	4.7911..$\times 10^{10}$	210.12	1.2883..$\times 10^{19}$

Table 2 Data for a neutron star and a black hole

Name	R_0 [m]	M [kg]	ρ_V [kg/m^3]	$g(R_0)$]m/s^2]	$End_g(R_0)$ [J/m^3]	$Vmd(R_0)$ [kg/m^3]	$End_{QV}(R_0)$ [J/m^3]
Neutron s.	19300	3.3876,$\times 10^{31}$	1,2871.$\times 10^{11}$	6.944.$\times 10^{11}$	2.8775..$\times 10^{32}$	2.1453..$\times 10^{16}$	1.02735..$\times 10^{33}$
Black h.	2878,.	3.3876,$\times 10^{31}$	3.8755.$\times 10^{19}$	3.121.$\times 10^{13}$	5.8107.$\times 10^{35}$	6.4659..$\times 10^{18}$	5.81076..$\times 10^{35}$

Comments to the data in Table 1

The last four functions, the surface gravitational field, the energy density of this field $End_g(R_0)$, the virtual mass density $Vmd(R_0)$ and the energy virtual density of QV i.e., $End_{QV}(R_0)$ are all circular symmetric functions of a radius r. The Table 1 presents the peak values at $r = R_0$ at the surface of the sphere. The cross-section of the virtual mass density $Vmd(r)$ defined by the Eq.(A4) is displayed in Fig.(1). The presented mass densities of all spheres of the solar system are simplified to be uniform and in the case of the Earth we get the surface g-field 9.0392.. [m/s^2] in comparison to 9.81.. [m/s^2]. The mass density of the Earth is the largest one 5941.7 [kg/m^3]. For all spheres in the Table 1 the peak value of the virtual mass density is

$$Vmd\left(R_0\right) = \frac{\rho_V}{6} \ \left[\text{kg/m}^3\right] \tag{14}$$

This equation shows, that for a given baryonic mass density $\rho_V(R_0)$ the virtual mass density is the same for all radii R_0, for example for a sphere of radius 1 meter is the same as for Earth.

Total energies obtained by integrating energy densities

The total energy of the gravitational field (A2) is given by the integral

$$E_g = \int_0^{R_0} 4\pi r^2 \frac{GM^2}{8\pi}\frac{r^2}{R_0^6} dr + \int_{R_0}^{\infty} 4\pi r^2 \frac{GM^2}{8\pi}\frac{1}{r^4} dr = \frac{6}{10}\frac{GM^2}{R_0} = \frac{6}{10}\phi\left(\mathrm{R}_0\right)\mathrm{M} \ [\mathrm{J}] \tag{15}$$

The inside integral yields 1/10 and the outside integral 5/10'

The total virtual mass is given by the integrals of the mass density (A4)

$$"M_{QV} = \int_0^{R_0} 4\pi r^2 \frac{M}{4\pi}\frac{r^2}{R_0^5}\frac{1}{\left(1+r^2/R_0^2\right)} dr + \lim_{R_1\to\infty}\int_{R_0}^{R_1} 4\pi r^2 \frac{M}{8\pi r^3} dr =$$

$$\mathrm{M}\left(\frac{\pi}{4}-\frac{2}{3}\right) + \frac{M}{2}\lim_{R_1\leftrightarrow\infty}\left[\log\left(\frac{R_1}{R_0}\right)\right] \tag{16}$$

We used $\int_0^1 \frac{x^4}{1+x^2} dx = \frac{\pi}{4} - \frac{2}{3} = 0.11873..$ Evidently, the second integral is divergent. For example for $R_1 = 100R_0$, the second integral equals M. This result suggests that in the Newton's law, the proportionality to $1/\mathrm{r}^2$ should by replaced by $1/\left(r^{2+\alpha}\right)$ or $e^{-\alpha r}/\left(r^2\right)$. We do not intend to study this problem in this paper. The total energy is given by $M_{QV}\, c^2$. This energy is several orders of magnitude bigger in comparison to the energy of the gravitational field. The ratio equals $c^2/\phi(R_0)$ is of the order 10^{10}.

Comments to the data in Table 2

The data in Table 2 have been calculated using the same formulae as in Table 1, i.e., assuming linear addition of gravitational fields. Actually the problem of addition of strong collinear fields is not solved. Certainly, there is a maximum possible value of a gravitational field. In Table 2 we read formidable values 6.944×10^{11} [m/s^2] at the surface of a neutron star and $3.121.\times10^{13}$ at the surface of a black hole. The above values should be classified as approximate. Having in mind the formula of addition of velocities

$$c_{1+2} = \frac{c_1 + c_2}{1 + \frac{c_1 c_2}{c^2}} \tag{17}$$

We could propose a following formulae for addition of collinear gravitational fields

$$g_{1+2} = \frac{g_1 + g_2}{1 + \frac{g_1 g_2}{g_{\max}^2}} \tag{18}$$

It was shown in [4] that the formula (16) should be modified by addition of higher order terms.

The ratio of gravitational to virtual energy density

The ratio of gravitational to virtual energy density at the surface of a circular body of radius R_0 and uniform mass density ρ_V is

$$ratio = \frac{|\phi(R_0)|}{c^2} = \frac{0.5\gamma|g|^2}{Vmd \times c^2} = \frac{\rho_V R_0^2}{3\gamma c^2} = 3.1100723 \times 10^{-27} \times \rho_V R_0^2$$

For the Earth we get *ratio=6.994866*¥10^{-10}. The virtual energy density is more than 10^9 larger w.r.t. the gravitational field energy density. We have a ***huge amplification***. For the black hole the ratio equals 1 (see Table 2) and both densities have a value 5.81…¥10^{35} [J/m^3] in comparison to 1.88…¥10^{13} for the Earth.

Unisotropic aureola

The energy densities defined by (5) and (7) are circular symmetric functions. In a previous paper, the author presented a hypothesis about the nature of gravitational forces [5]. They are explained as the result of unisotropic energy exchange between QV and a particular body. This unisotropy is induced by an unisotropic energy aureole around the body. Let us study this unisotropy due by the deformation of the aureole of Earth by the gravitation field of the Sun. Modern astronomy is able to calculate and measure the orbit of Earth around the Sun with high accuracy. Here we study only the deformation of the Earth aurela by the gravity of the Sun. We use the following assumptions:

1. At a certain moment the center of the Earth is located at the origin of Cartesian coordinates (x,y,x) and the Sun is located at x_{Sun}=1.49587..×10^{11}[m] yielding the value of gravity at the Earth g_{Sun}= 5.9318..×10^{-3} [m/s^2] about 6×10^{-4} of the selfgravity at the surface of Earth.
2. The Earth is transparent to the gravity of the Sun (no shilding).
3. The mean force of attraction of the Earth by the gravity of Sun is

$$F = \frac{GM_{Sun}M_{Earth}}{x_{Sun}} = 3.546... \times 10^{22} \ [\mathrm{N}] \tag{19}$$

Let us arbitrary divide this force by a surface $4\pi R_0^2$ getting a pressure

$$P = \frac{F}{4\pi R_0^2} = 7.005.. \times 10^7 \ \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right] \text{ or } \left[\frac{\mathrm{J}}{\mathrm{m}^3}\right] \tag{20}$$

Assuming that the gravity of the Sun is approximated the same of both sides of the earth we have:

$$\text{Gravity at the Sun side is } g_-(R_0) = g_{Earth} - g_{Sun} \tag{21}$$

$$\text{Gravity at opposite side is } g_+(R_0) = g_{Earth} + g_{Sun} \tag{22}$$

The difference in energy densities

$$\Delta End_g = 0.5\gamma\left[g_+^2 - g_-^2\right] = 2\gamma g_{Sun} g_{Earth} = 7.005.. \times 10^7 \ \left[\mathrm{J/m^3}\right] \tag{23}$$

i.e., equals exactly the pressure defined by (19). The virtual mass density of the Sun at the Earth is

$$Vmd_{SE} = \frac{M_{Sun}}{8\pi d^3} = 2.364... \times 10^{-5} \ \left[\mathrm{kg/m^3}\right] \tag{24}$$

And the corresponding virtual energy density is

$$Vmd_{SE} \times c^2 = 2.124... \times 10^{18} \left[\text{J/m}^3\right] \tag{25}$$

A comment about the difference between the gravitational field and the virtual energy density field

The gravitational field is a vector field. Two opposite fields cancel but do not annihilate. For example they cancel at so called Lagrange points. Differently, the virtual energy density field is a scalar field. It is responsible for the lowering of the speed of light by gravitation. It is reasonable to assume that this field changes the electromagnetic properties of the QV represented by the impedance and permittivity of free space. The eventual experimental validation should apply the measurements of the speed of light along and perpendicular to a uniform gravitational field. The result should be the same. Note that at the surface of Earth the gravitational field is not uniform.

Concluding:

1. The energy density of the *g*- field is lower at the anti-Sun side, Therefore the pressure defined by (20) and (23) should be directed in the anti-Sun direction. Evidently, it is directed towards the Sun. In the paper [5] this discrepancy has been explained that the gravitational force is the result of anisotropic energy exchange between QV and baryonic matter.
2. The coincidence of Eqs (20) and (23) shows that the gravitational force is proportional to the product of Earth and Sun gravity.
3. The derivations apply to a nonorbiting Earth. In the case of orbiting Earth, the gravitational force is almost completely cancelled by the opposite inertial force. We say "almost", since there is a small component tangential to the orbit [6].

Appendix: The gravitational field of a sphere , potential function and energy densities

Consider a sphere of radius R_0 and a total mass M and a uniform mass density $\rho_V = M / \left((4/3)\pi R_0^3 \right) \left[\text{kg/m}^3\right]$ corresponding to an abstract surface mass density $\rho_S = M / \left(4\pi R^2 \right) \left[\text{kg/m}^2\right]$ The gravitational field of this sphere is given by

$$g(r) = \begin{cases} -GM \dfrac{r}{R_0^3}\overline{r} = -\dfrac{\rho_S}{\gamma}\dfrac{r}{R_0}\overline{r} = -\dfrac{\rho_V}{3\gamma} r\overline{r} \ \text{ if } \ r < R_0 \\ -\dfrac{GM}{R_0^2}\overline{r} = -\dfrac{\rho_S}{\gamma}\overline{r} = -\dfrac{\rho_V}{3\gamma} r\overline{r} \ \text{ if } \ r = R_0 \\ -\dfrac{GM}{r^2}\overline{r} = -\dfrac{\rho_S}{\gamma}\dfrac{R_0^2}{r^2}\overline{r} = -\dfrac{\rho_V}{3\gamma}\dfrac{R_0^2}{r^2}\overline{r} \ \text{ if } \ r > R_0 \end{cases} \tag{A1}$$

$G \left[\text{m}^3 / \left(kgs^2\right)\right]$ is the gravitational constant and $\gamma = 1/\left(4\pi \text{G}\right) \left[\text{kgs}^2 / \text{m}^3\right]$. The reciprocal gravitational constant $\overline{r}$ is a unit versor. The energy density of the gravitational field is negative and given by

$$End_g\left(r\right) = -0.5\gamma \left|g\left(r\right)\right|^2 \tag{A2}$$

Note that we have a lowering by the above amount the positive energy density given by (xx). The potential function of the sphere of a radius R_0 and mass *M is*

$$\phi(r)=\begin{cases}\dfrac{-GM}{2R_0}\left(1+\dfrac{r^2}{R_0^2}\right) & \text{if } r<R_0\\ \dfrac{-GM}{r} & \text{if } r\ge R_0\end{cases} \tag{A3}$$

Let us remind that in the case of a point mass the potential function has the form as in the second line of (A4)

$$\phi(r)=-\frac{GM}{r}=-\frac{\rho_S}{\gamma}\frac{R_0^2}{r}=-\frac{\rho_V}{3\gamma}\frac{R_0^3}{r}$$

Using the general formula (A4) the virtual mass density of QV is

$$Vmd_{QV}(r)=\frac{End_g(r)}{\phi(r)}=\begin{cases}\dfrac{M}{4\pi}\dfrac{r^2}{R_0^5}\left(1+\dfrac{r^2}{R_0^2}\right)^{-1} & \text{if } r<R_0\\ \dfrac{M}{8\pi R_0^3} & \text{if } r=R_0\\ \dfrac{M}{8\pi r^3} & \text{if } r>R_0\end{cases} \tag{A4}$$

Note, that for $r=R_0$ Vmd_{QV} equals 1/6 of ρ_{V}. The corresponding energy density

$$End_{QV}=\left|Vmd_{QV}\right|c^2 \tag{A5}$$

is also a circular symmetric function. In this form it represents the local lowering of the Planck energy density (see (10).

The total energy of the gravitational field and total mass of the equivalent mass density

The total energy of the gravitational field (A2) is given by the integral

$$E_g=\int_0^{R_0}4\pi r^2\frac{GM^2}{8\pi}\frac{r^2}{R_0^6}dr+\int_{R_0}^{\infty}4\pi r^2\frac{GM^2}{8\pi}\frac{1}{r^4}dr=\frac{6}{10}\frac{GM^2}{R_0}\ [\mathrm{J}] \tag{A6}$$

The total "mass" (the equivalent mass) is given by the integrals of the mass density (A4)

$$"M_{QV}=\int_0^{R_0}4\pi r^2\frac{M}{4\pi}\frac{r^2}{R_0^5}\frac{1}{\left(1+r^2/R_0^2\right)}dr+\lim_{R_1\to\infty}\int_{R_0}^{R_1}4\pi r^2\frac{M}{8\pi r^3}dr=M\left(\frac{\pi}{4}-\frac{2}{3}\right)+\frac{M}{2}\lim_{R_1\to\infty}\left\{\log\left(\frac{R_1}{R_0}\right)\right\} \tag{A7}$$

We used $\int_0^1\frac{x^4}{1+x^2}dx=\frac{\pi}{4}-\frac{2}{3}=0.11873..$

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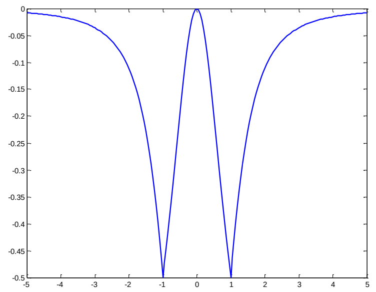

x

Fig.1 One dimensional cross-section of the equivalent mass density *Vmd*(*r*) defined by Eq.(A4). Horizontal scale $x=r/R_0$. The range of *x* should be 5 to 5 and not -5 to 5. Vertical scale from -0 to 1 is normalized by $M/\left(8\pi R_0^2\right)$.

In this figure negative sign of the aureole is applied. The same form has the cross-section of the energy density $\rho_{QV}(r)c^2$.